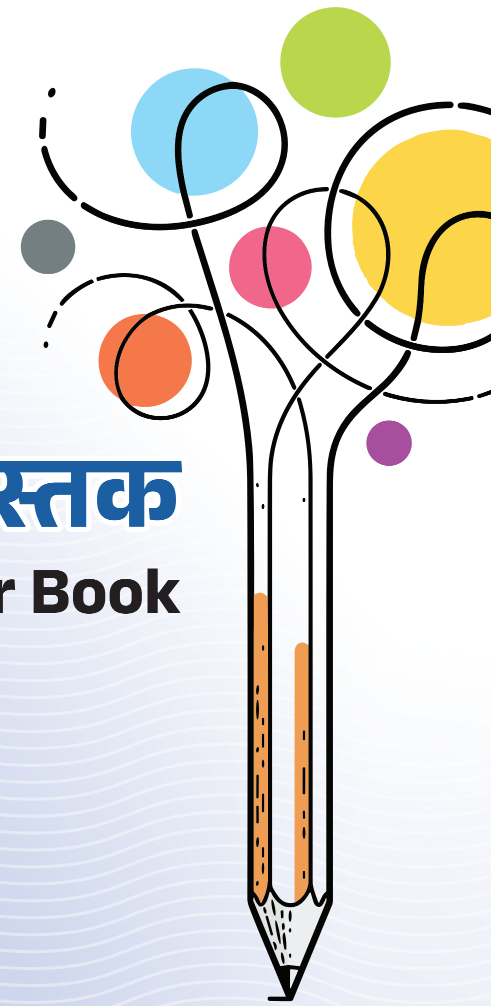


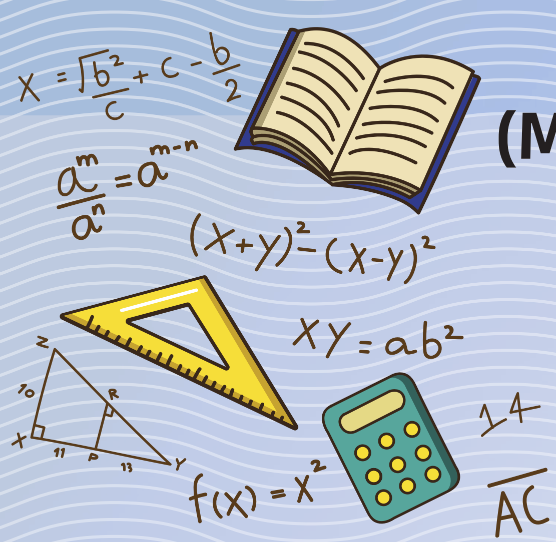
प्रश्न बैंक-सह-उत्तर पुस्तक

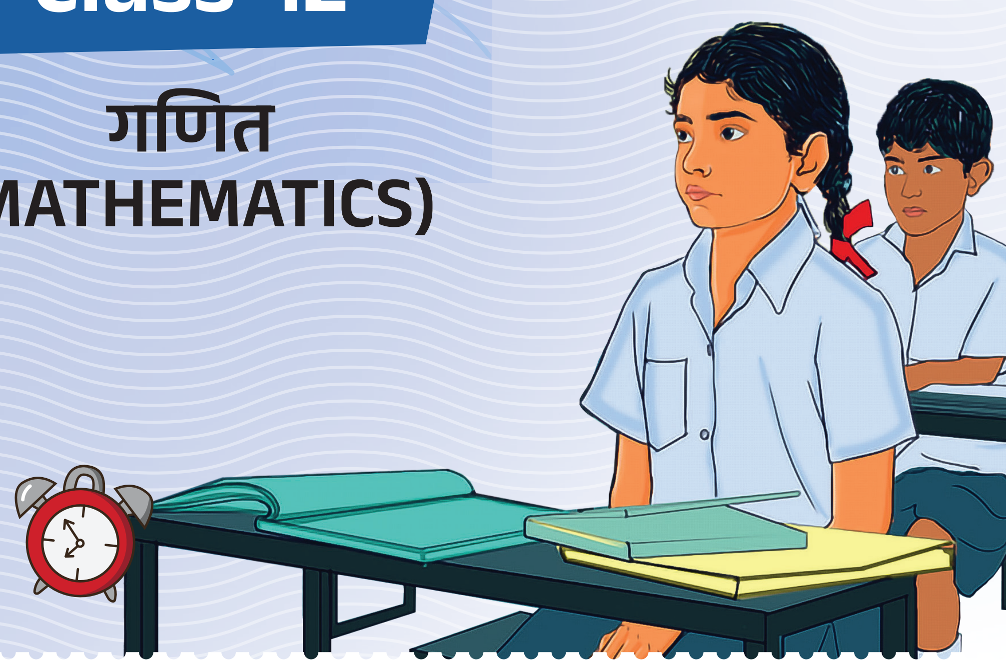
Question Bank–Cum–Answer Book

2023

Class-12

गणित
(MATHEMATICS)





झारखण्ड शैक्षिक अनुसंधान एवं प्रशिक्षण परिषद्, राँची

Jharkhand Council of Educational Research and Training, Ranchi

प्रश्न बैंक–सह–उत्तर पुस्तक

Question Bank-Cum-Answer Book

Class - 12

गणित

Mathematics

2023

झारखंड शैक्षिक अनुसंधान एवं प्रशिक्षण परिषद्, राँची

Jharkhand Council of Educational Research and Training, Ranchi

प्रथम संस्करण : माह अगस्त 2023, श्रावण विक्रम संवत् 2080

झारखंड शैक्षिक अनुसंधान एवं प्रशिक्षण परिषद्, राँची, झारखंड द्वारा प्रकाशित

प्राक्कथन

बच्चों के लिए निर्धारित अधिगम प्रतिफल प्राप्त करने का मार्ग सरल एवं सुगम होना आवश्यक है। इस उद्देश्य को ध्यान में रखते हुए झारखंड शैक्षिक अनुसंधान एवं प्रशिक्षण परिषद्, राँची, झारखंड के द्वारा कक्षा 12 के सभी विषयों के लिए प्रश्न बैंक–सह–उत्तर पुस्तक का निर्माण बच्चों के अधिगम कौशल को सुगमतापूर्वक विकसित करने एवं झारखंड अधिविद्य परिषद् द्वारा आयोजित वार्षिक इंटरमीडिएट परीक्षा के लिए उन्हें तैयार करने के उद्देश्य से किया गया है। इस प्रश्न बैंक–सह–उत्तर पुस्तक में सरल भाषा एवं रुचिकर ढंग से विषय–वस्तु को स्पष्ट करते हुए प्रश्नोत्तर दिए गए हैं। इस प्रश्न बैंक–सह–उत्तर पुस्तक के माध्यम से बच्चों में न केवल ज्ञानजन्य प्रतिभा का विकास होगा बल्कि आज के इस प्रतियोगिता के दौर में भी वे अनुकूल सफलता पाएंगे। हमारे प्रयत्न की सफलता इस बात पर निर्भर करती है कि विद्यालय के शिक्षकवृन्द बच्चों की कल्पनाओं के साथ कितना जुड़ पाते हैं और विभिन्न प्रकार के प्रश्नोत्तरों को सीखने–सिखाने के दौरान अपने अनुभवों के साथ–साथ बच्चों के विचारों के साथ कैसे सामंजस्य बनाते हैं।

इस प्रश्न बैंक–सह–उत्तर पुस्तक में झारखंड अधिविद्य परिषद् द्वारा आयोजित वार्षिक इंटरमीडिएट परीक्षा में पूछे जाने वाले प्रश्नों के विविध प्रकारों यथा– बहुवैकल्पिक, अतिलघु उत्तरीय, लघु उत्तरीय एवं दीर्घ उत्तरीय प्रश्न आदि के अंतर्गत पर्याप्त मात्रा में प्रश्नोत्तर समाहित किए गए हैं ताकि इसके अध्ययन से छात्रों में ना केवल विषय–वस्तु की समझ विकसित हो बल्कि उन्हें सीखने के प्रतिफल की भी प्राप्ति हो, साथ ही वार्षिक इंटरमीडिएट परीक्षा के लिए उनकी अच्छी तैयारी हो सके और वे परीक्षा में बेहतर प्रदर्शन करते हुए सफलता प्राप्त कर सकें।

अंत में मैं इन पुस्तकों के लेखकों के प्रति आभार व्यक्त करता हूँ।

शुभकामनाओं के साथ।

के० रवि कुमार भा.प्र.से.
सचिव
स्कूली शिक्षा एवं साक्षरता विभाग, झारखण्ड

भूमिका

प्रिय शिक्षक एवं विद्यार्थी,

जोहार !

हमें कक्षा 12 के विभिन्न विषयों के प्रश्न बैंक–सह–उत्तर पुस्तक से आपका परिचय कराने में प्रसन्नता हो रही है। इस प्रश्न बैंक–सह–उत्तर पुस्तक में झारखण्ड शैक्षिक अनुसन्धान एवं प्रशिक्षण परिषद्, राँची द्वारा प्रकाशित पाठ्यपुस्तकों के विषयवार एवं अध्यायवार अधिगम बिन्दुओं को समायोजित करते हुए झारखण्ड अधिविद्य परिषद् द्वारा आयोजित वार्षिक इंटरमीडिएट परीक्षा में पूछे जानेवाले प्रश्नों के विविध प्रकारों के अंतर्गत पर्याप्त मात्रा में प्रश्नों का समावेश किया गया है। इस विषय आधारित प्रश्न बैंक–सह–उत्तर पुस्तक के निर्माण का उद्देश्य शिक्षण अधिगम प्रक्रिया को और अधिक रुचिकर, सरल एवं प्रभावशाली बनाना तथा विद्यार्थियों को वार्षिक इंटरमीडिएट परीक्षा की तैयारियों में सहयोग प्रदान करना है, जिससे सकारात्मक रूप से छात्रों को सीखने के प्रतिफल प्राप्त हों और वार्षिक इंटरमीडिएट परीक्षा में वे बेहतर प्रदर्शन कर सकें। राज्य के विभिन्न जिलों से चयनित अनुभवी शिक्षकों के द्वारा इस प्रश्न बैंक–सह–उत्तर पुस्तक का निर्माण किया गया है।

इस प्रश्न बैंक–सह–उत्तर पुस्तक की प्रमुख विशेषताएँ यह है कि इनमें प्रश्नों के उत्तर को सरल भाषा में प्रस्तुत करते हुए वैचारिक समझ (Conceptual Understanding) विकसित करने पर जोर दिया गया है। साथ ही इन पुस्तकों में झारखण्ड अधिविद्य परिषद् द्वारा आयोजित वार्षिक इंटरमीडिएट परीक्षा – 2023 के प्रश्नोत्तर को भी समाहित किया गया है। इन पुस्तकों के माध्यम से न केवल विद्यार्थियों की प्रतिभा में निखार आएगा बल्कि वर्तमान समय के प्रतियोगिताओं के इस दौर में वे अनुकूल एवं अपेक्षित सफलता प्राप्त करने में भी सक्षम हो सकेंगे। आशा है कि यह प्रश्न बैंक–सह–उत्तर पुस्तक आपको पसंद आएगी एवं आपके लिए उपयोगी सिद्ध होगी।

शुभकामनाओं के साथ।

किरण कुमारी पासी भा.प्र.से.
निदेशक
झारखण्ड शैक्षिक अनुसंधान एवं प्रशिक्षण परिषद्
राँची, झारखण्ड

पाठकों से अनुरोध

इस प्रश्न बैंक–सह–उत्तर पुस्तक के निर्माण में काफी सावधानियाँ बरती गई है। इसके बावजूद यदि किसी प्रकार की अशुद्धियाँ मिले या कोई सुझाव हो तो इस email ID :- jcertquestionbank@gmail.com पर सूचित करें, ताकि अगले मुद्रण में इसे शुद्ध रूप से प्रस्तुत किया जा सके।

प्रश्न बैंक–सह–उत्तर पुस्तक निर्माण समिति

मुख्य संरक्षक

श्री के० रवि कुमार (भा.प्र.से.)
सचिव
स्कूली शिक्षा एवं साक्षरता विभाग, झारखण्ड

संरक्षक

श्रीमती किरण कुमारी पासी (भा.प्र.से.)
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अवधारणा एवं मार्गदर्शन

श्री मुकुंद दास उपनिदेशक (प्र.) झारखण्ड शैक्षिक अनुसन्धान एवं प्रशिक्षण परिषद्, राँची	**श्री बाँके बिहारी सिंह** सहायक निदेशक (अ.) झारखण्ड शैक्षिक अनुसन्धान एवं प्रशिक्षण परिषद्, राँची	**श्री मसुदी टुडू** सहायक निदेशक (अ.) झारखण्ड शैक्षिक अनुसन्धान एवं प्रशिक्षण परिषद्, राँची

समन्वय एवं निर्देशन

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सहयोग

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संकाय सदस्य, जे.सी.ई.आर.टी., राँची
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विषय सूची

Chapter: 1 Relations and Functions
सम्बन्ध एवं फलन

बहुविकल्पीय प्रश्न (MCQ)

1 Marks Question:-

1. If A={1,2,3,4} and R be a relation defined on A as R={(1,2) (2,1), (3,4), (4,3)} then R is –

यदि A={1,2,3,4}और R समुच्चय A पर R={(1,2), (2,1), (3,4), (4,3)}परिभाषित एक संबंध है तो R है–

(a) Reflexive / स्वतुल्य
(b) Symmetric / सममित
(c) Transitive / संक्रामक
(d) None of them / इनमें से कोई नहीं।

2. A relation R in a set A is said to be an equivalence relation if and only if–

एक संबंध R किसी समुच्चय A पर तुल्यता संबंध कहलाता है, यदि और केवल यदि–

(a) Only Reflexive / केवल स्वतुल्य
(b) Only Symmetric / केवल सममित
(c) Only Transitive / केवल संक्रामक
(d) All of them ie, Reflexive, Symmetric and Transitive उपरोक्त सभी। अर्थात् स्वतुल्य, सममित एवं संक्रामक

3. A relation R in a set A is a subset of–

किसी समुच्चय A पर एक संबंध R उपसमुच्चय है–

(a) Set A / समुच्चय A का।
(b) Cartesian product of A ie, AxA / कार्तीय गुणन AxA का
(c) ϕ / ϕ
(d) None of them / इनमें से कोई नहीं।

4. If A={1,2,3,4} and R be a relation defined on A as R={(2,2), (3,3), (1,2), (2,1)} then R is–

यदि A={1,2,3,4} तथा R समुच्चय A पर परिभाषित एक संबंध R={(2,2), (3,3), (1,2), (2,1)} है तो R है–

(a) Reflexive / स्वतुल्य
(b) Symmetric / सममित
(c) Transitive / संक्रामक
(d) None of them / इनमें से कोई नहीं।

5. If $A = \{x \in z: 0 \le x \le 6\}$ then A is–

यदि $A = \{x \in z: 0 \le x \le 6\}$ तो A है–

(a) {1, 2, 3}
(b) {0, 2, 4, 6}
(c) {0, 1, 2, 3, 4, 5, 6}
(d) None of them / इनमें से कोई नहीं।

6. If $A = \{x \in \mathbf{Z} : 2 \le x \le 6\}$ then A is–

यदि $A = \{x \in \mathbf{Z} : 2 \le x \le 6\}$ तो A है–

(a) {2,4,6}
(b) {2,3,4,5,6}
(c) ϕ
(d) {3,5}

7. If f : R → R, given by f(x) = 4x+3 then f^{-1} (f inverse) is–

यदि f : R → R, में परिभाषित फलन f(x) = 4x+3 है तो f^{-1} है–

(a) $f^{-1}(x) = \frac{x-3}{4}$
(b) $f^{-1}(x)$ does not exist / $f^{-1}(x)$ प्राप्त नहीं हैं।
(c) $f^{-1}(x) = \frac{x-4}{3}$
(d) None of them /इनमें से कोई नहीं।

8. If f : R → R be a function such that f(x) = 2x+1, then inverse of f ie, f^{-1} is –

यदि f : R → R में परिभाषित एक फलन f(x) = 2x+1 है, तो f^{-1} है। –

(a) $f^{-1}(x) = \frac{x-1}{2}$
(b) $f^{-1}(x) = \frac{x-2}{3}$
(c) $f^{-1}(x)$ does not exist / $f^{-1}(x)$ **प्राप्त नहीं होता है।**
(d) None of them / इनमें से कोई नहीं।

9. If f(x) = $\sin^2 x$ and g(x) = x then (fog) (x) is–

यदि f(x) = $\sin^2 x$ और g(x) = x हैं तो (fog) (x) है–

(a) sin x
(b) $\sin^2 x$
(c) $\sin x^2$
(d) None of them / इनमें से काई नहीं।

10. If f(x) = sin x and g(x) = x^2 then (fog) is–

यदि f(x) = sin x और g(x) = x^2 है तो (fog) हैं।–

(a) sin x
(b) $\sin x^2$
(c) $\sin^2 x$
(d) None of them / इनमें से काई नहीं।

11. If $f(x) = e^x$ and $g(x) = \log x$ then (gof) is –

यदि $f(x) = e^x$ और $g(x) = \log x$ है तो (gof) हैं।–

(a) $e^{\log x}$

(b) $\log e^x$

(c) $\log x.e^x$

(d) None of them / इनमें से काई नहीं।

12. If $f(x) = \log x$ and $g(x) = \sin x$ then (gof) is–

यदि $f(x) = \log x$ और $g(x) = \sin x$ है तो (gof) हैं।–

(a) $\sin(\log x)$

(b) $\log(\sin x)$

(c) $\log x.\sin x$

(d) None of them / इनमें से काई नहीं।

13. If binary operation '*' is defined as a*b = ab+1 then 2*4 is equal to–

यदि एक द्विआधारी संक्रिया *, a*b = ab+1 द्वारा परिभाषित है तो 2*4 का मान है।–

(a) 9

(b) 7

(c) 6

(d) 1

14. A Binary operation 'o' is defined as $aob = \frac{(ab)^2}{2}$ then 1o4 is equal to –

एक द्विआधारी संक्रिया 'o', $aob = \frac{(ab)^2}{2}$ द्वारा परिभाषित है तो 1o4 का मान हैं।–

(a) 8

(b) 4

(c) 2

(d) 16

15. If $f : R \to R$ be a function defined by $f(x) = 2x - 3, \forall x \in R$ then f^{-1} is –

यदि $f : R \to R$ एक फलन $f(x) = 2x - 3, \forall x \in R$ द्वारा परिभाषित है तो f^{-1} है। –

(a) $f^{-1}(x) = \frac{x+3}{2}$

(b) $f^{-1}(x) = \frac{x+2}{3}$

(c) $f^{-1}(x) = 3x - 2$

(d) f^{-1} does not exist / f^{-1} प्राप्त नहीं होता है।

16. If $f : R \to R$, is defined by $f(x) = x^2+2$ then (fof) is –

यदि $f : R \to R$, $f(x) = x^2+2$ द्वारा परिभाषित है तो (fof) है। –

(a) $x^4 + 4x^2 + 6$

(b) $4x^2 + 6$

(c) $x^4 + 6$

(d) $x^4 + 6x^2 + 4$

17. If $f : R \to R$ is defined by $f(x) = (x+2)$ then (fof) is –

यदि $f : R \to R$, $f(x) = (x+2)$द्वारा परिभाषित है तो (fof) है।–

(a) x + 4

(b) x + 6

(c) x + 8

(d) x + 10

18. If R be the relation in the set N given by R={(a,b) : a = b - 2 , b > 6}, then -

यदि समुच्चय N पर R एक संबंध है जो R={(a,b) : a = b - 2 , b > 6} द्वारा परिभाषित है तो –

(a) $(2,4) \in R$

(b) $(3,8) \in R$

(c) $(6,8) \in R$

(d) $(8,7) \in R$

19. If R be the relation in the set N given by R={(a,b) : b = 2a , a > 4 }, then -

यदि समुच्चय N पर R एक संबंध है जो R = {(a,b) : b = 2a , a > 4 }, द्वारा परिभाषित है तो –

(a) $(5,10) \in R$

(b) $(10,5) \in R$

(c) $(4,8) \in R$

(d) $(4,10) \in R$

20. If R be the set of all real numbers , then the function $f : R \to R$ defined by $f(x) = |x|$ is -

यदि R वास्तविक संख्याओं का समुच्चय है तथा एक फलन $f : R \to R$, $f(x) = |x|$ द्वारा परिभाषित है तो f है–

(a) one -one only / केवल एकैक

(b) onto only / केवल आच्छादक

(c) neither one - one nor onto / न एकैक न आच्छादक

(d) both one -one and onto / एकैक एवं आच्छादक दोनो

21. A binary operation $* : A \times A \to A$ is said to be associative if –

एक द्विआधारी संक्रिया $* : A \times A \to A$ को साहचर्य कहा जाता है यदि –

(a) (a * b) * c = a * (b * c)

(b) (a * b) * c = (a * c) * b

(c) (a * b) * c = b * (a * c)

(d) None of them / इनमें से कोई नहीं।

22. **A binary operation * on the set X is called commutative if – (for every $a,b \in X$)**

एक द्विआधारी संक्रिया * किसी समुच्चय X पर क्रम विनिमेय कहलाता है यदि –

(a) $a*b = b*a$
(b) $a*b = a*b$
(c) $b*a = b*a$
(d) None of them / इनमें से कोई नहीं।

अति लघु उत्तरीय प्रश्न (Very Short Question)

2 Marks Question:-

1. **Is the function $f:N \rightarrow N$, surjective (onto) where $f(x)=2x+3$.**

क्या $f:N \rightarrow N$, आच्छादक फलन है? जबकि $f(x)=2x+3$.

2. **If a function $f:R \rightarrow R$ defined by $f(x) = |x|$, $x \in R$ then examine whether the function is one-one or many one.**

फलन $f : R \rightarrow R$ को एकैक या बहुएक के लिए जाँचें; जबकि $f(x) = |x|, x \in R$

3. **Examine whether the function $f:R \rightarrow R$, defined by $f(x) = x^3, x \in R$ is one-one?**

क्या फलन $f: R \rightarrow R$ एकैक फलन है, जहाँ $f(x) = x^3, x \in R$?

4. **Examine whether the function $f:N \rightarrow N$, where $f(x)=3x, x \in N$ is onto?**

क्या फलन $f:N \rightarrow N$, आच्छादक है, जहाँ $f(x)=3x$, $x \in N$?

लघु उत्तरीय प्रश्न (Short Question)

3 Marks Question:-

1. **If $f:R \rightarrow R$, $g:R \rightarrow R$ are two functions such that $f(x) = x^2$ and $g(x) = x^3$ then find the functions $(fog)(x)$ and $(gof)(x)$. Are (fog) and (gof) equal functions?**

यदि $f:R \rightarrow R$, $g:R \rightarrow R$ दो फलन है, जहाँ $f(x) = x^2$ और $g(x) = x^3$ तो फलन (fog) तथा (gof) ज्ञात करें। क्या (fog) और (gof) बराबर फलन है?

2. **If $f:R \rightarrow R$, be a function defined by $f(x)=2x+7$, $x \in R$, define $f^{-1}:R \rightarrow R$ Also find the value of $f^{-1}(3)$.**

यदि $f:R \rightarrow R$ एक फलन है जहाँ $f(x)=2x+7$, $x \in R$, तो $f^{-1}:R \rightarrow R$ परिभाषित करें, एवं $f^{-1}(3)$ का मान ज्ञात करे।

3. **Prove that the function $f:R \rightarrow R$ is an one-one onto function, where $f(x)=2x$, $x \in R$.**

सिद्ध करें कि फलन $f:R \rightarrow R$,जहाँ $f(x)=2x$, $x \in R$ एक one-one onto फलन है।

4. **If $f:R \rightarrow R$ is defined by $f(x) = x^2-3x+2$ then find the value of $f(f(x)) = ?$**

यदि $f:R \rightarrow R$ एक फलन है, जहाँ $f(x) = x^2-3x+2$, $x \in R$ तो $f(f(x))$ का मान निकालें ?

दीर्घ उत्तरीय प्रश्न (Long Question)

5 Marks Question:-

1. **Let us suppose set $A=\{1,2,3\}$, $B=\{4,5,6,7\}$ and $f=\{(1,4), (2,5), (3,6)\}$. Prove that f is a function from A to B, which is one-one but not onto?**

माना कि $A=\{1,2,3\}$, $B=\{4,5,6,7\}$ और $f=\{(1,4), (2,5), (3,6)\}$, तो सिद्ध करें कि f , A से B में एक फलन है जो one-one तो है परन्तु onto नहीं ?

2. **Suppose the function $f:N \rightarrow N$ be defined as follows–**

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{; when n is odd} \\ \frac{n}{2} & \text{; when n is even} \end{cases}$$

then verify whether f is bijective or not?

यदि फलन $f:N \rightarrow N$ निम्नलिखित रूप से परिभाषित है।

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{; जब n विषम संख्या हैं।} \\ \frac{n}{2} & \text{; जब n सम संख्या हैं।} \end{cases}$$

तो जाँच कर बताएँ कि f, bijective है कि नहीं?

3. **Prove that the function $f:R \rightarrow R$, defined by**

$$f(x) = \begin{cases} 1 \;; & x>0 \\ 0 \;; & x=0 \\ -1 \;; & x<0 \end{cases}$$

is neither one-one nor onto function?

सिद्ध कीजिए कि $f:R \rightarrow R$,

$$f(x) = \begin{cases} 1 \;; \text{ यदि } & x > 0 \\ 0 \;; \text{ यदि } & x = 0 \\ -1 \;; \text{ यदि } & x < 0 \end{cases}$$

द्वारा प्रदत्त चिह्न फलन न तो एकैकी है और न ही आच्छादक है।

Answer:"Relations and Functions"

MCQ Solutions:

1. (b)	9. (b)	17. (a)
2. (d)	10. (b)	18. (c)
3. (b)	11. (b)	19. (a)
4. (b)	12. (a)	20. (c)
5. (c)	13. (a)	21. (a)
6. (b)	14. (a)	22. (a)
7. (a)	15. (a)	
8. (a)	16. (a)	

Very Short Question (2 Marks Solution)

1. f:N → N such that f(x) = 2x + 3, x ∈ N

Let y = 2x + 3

$\Rightarrow 2x = y - 3$

$\Rightarrow x = \frac{y-3}{2}$

if we put y=4 then $x = \frac{4-3}{2} = \frac{1}{2} \notin N$

Thus, $4 \in N$ has no pre-image in N.

⇒ f is not onto.

2. f:R → R s.t; f(x) = |x|, x ∈ R

clearly f(-1) = |-1|

= 1

and f(1) = |1|=1

i.e. f(-1) = 1= f(1)

but -1 ≠ 1

Thus two different elements in R have the same image.

⇒ f is not one-one.

i.e. f is many one.

3. f:R → R, such that $f(x)=x^3$; x ∈ R

$Let\ \ f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow (x_1^3 - x_2^3) = 0$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

$\Rightarrow (x_1 - x_2)\left[\left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3}{4}x_2^2\right] = 0$

$\therefore either$

$x_1 - x_2 = 0\ or\ \left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3}{4}x_2^2 = 0$

$but\ \left(x_1 + \frac{x_2}{2}\right)^2 + \frac{3}{4}x_2^2 \neq 0$

$\Rightarrow x_1 = x_2$

⇒ f is a one-one

4.

$f:N \longrightarrow N;\ f(x) = 3x,\ x \in N$

$Let\ y = 3x$

$\Rightarrow x = \frac{y}{3}$

$if\ we\ put\ y = 1\ then\ x = \frac{1}{3} \notin N$

Thus 1∈N has no pre-image in N

⇒ f is not onto

Short Question (3 Marks Solution)

1.

$f:R \longrightarrow R,\ s.t.,\ f(x) = x^2\ and$

$g:R \longrightarrow R,\ s.t.,\ g(x) = x^3$

$\therefore (fog)(x) = f[g(x)]$

$= f[x^3]$

$= (x^3)^2$

$\Rightarrow (fog)(x) = x^6 ---- (1)$

$Again,\ (gof)(x) = g[f(x)]$

$= g[x^2]$

$= [x^2]^3$

$\Rightarrow (gof)(x) = x^6 ---- (2)$

$from\ eq^{ns}\ 1\ and\ 2\ we\ get$

$fog = gof.$

2.

$f:R \longrightarrow R\ such\ that\ f(x) = 2x + 7,\ x \in R$

$then\ f^{-1}:R \longrightarrow R\ will\ exist$

$as\ f\ is\ an\ one - one\ onto\ function$

$Let\ y \in R\ be\ an\ image\ of\ x \in R$

$\therefore f(x) = y$

$\Rightarrow 2x + 7 = y$

$\Rightarrow 2x = y - 7$

$\Rightarrow x = \frac{y-7}{2}$

$as\ f(x) = 2x + 7$

$\Rightarrow f^{-1}(2x+7) = x$

$\Rightarrow f^{-1}(y) = \frac{y-7}{2}$

$or\ f^{-1}(x) = \frac{x-7}{2}$

$Thus\ f^{-1}:R \longrightarrow R\ such\ that\ f^{-1}(x) = \frac{x-7}{2};\ x \in R$

$hence,\ f^{-1}(3) = \frac{3-7}{2} = -2$

3. f : R→R such that f(x) = 2x, x∈ R

$Let\ f(x_1) = f(x_2); where\ x_1, x_2 \in R$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

Thus f is a one-one function over R

Again,

$Let\ y = 2x$

$\Rightarrow x = \frac{y}{2}$

$if\ we\ put\ y = 1\ then\ x = \frac{1}{2} \in R$

Thus, y = 1 ϵ R has a pre-image in R

⇒ f is an onto function

hence, we say that f is a one-one onto function.

4. $f{:}R \longrightarrow R\ such\ that$

$f(x) = x^2 - 3x + 2; x \in R$

$\because f(f(x)) = f[f(x)]$

$= f[x^2 - 3x + 2]$

$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

$= (x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2) -$

$(3x^2 - 9x + 6) + 2$

$\Rightarrow f[f(x)] = x^4 + 9x^2 + 4 - 6x^3$

$-12x + 4x^2 - 3x^2 + 9x - 6 + 2$

$= x^4 - 6x^3 + (9 + 4 - 3)x^2 + (-12 + 9)x$

$+(\cancel{4} - \cancel{6} + \cancel{2})$

$= x^4 - 6x^3 + 10x^2 - 3x$

$\Rightarrow f[f(x)] = x^4 - 6x^3 + 10x^2 - 3x.$

Long Question (5 Marks Solution)

1.

$f{:}A \longrightarrow B; where\ A = \{1,2,3\}, B = \{4,5,6,7\}$

$such\ that\ f = \{(1,4),(2,5),(3,6)\}$

$clearly, f(1) = 4$

$f(2) = 5$

$and\ f(3) = 6$

$\because$ *each* $x \in A$ *has unique image in B*

under the function f

$\therefore$ *f is a one – one functiion*

Again,

there exist $7 \in$ *codomain 'B' such that*

there does not exist any $a \in A$

ie, $7 \in B$ *is such an element which has no pre – image in set A*

$\Rightarrow$ *f is not onto*

hence, $f{:}A \longrightarrow B$ *is a function which is one – one but not onto*

2. $f{:}N \longrightarrow N\ such\ that,$

$$f(n) = \begin{cases} \frac{n+1}{2} & ; n = odd \\ \frac{n}{2} & ; n = even \end{cases}$$

∵ We have

f(1) $= \frac{1+1}{2}$, as n = 1 = odd

$= \frac{2}{2} = 1$

and f(2) $= \frac{2}{2}$, as n=2=even

= 1

⇒ f(1) = 1= f(2)

but 1 ≠ 2

ie, we get same image for two different elements.

Thus, f is not one-one

ie, f is many-one function

Again,

Let n ∈ N is an arbitrary element.

if n =odd ⇒ (2n-1) is odd

∴ f(2n-1) $= \frac{(2n-1)+1}{2}$

$= \frac{2n}{2}$

⇒ f(2n-1) = n

if n = even ⇒ 2n is even

∴ f(2n) $= \frac{2n}{2}$

⇒ f(2n) = n

Thus, for each n ∈ N there exist its pre-image in N

⇒ f is onto

hence, f is not one-one but onto function

ie, f is not bijective

3. f:R→R; such that

$$f(x) = \begin{cases} 1 & ; x>0 \\ 0 & ; x=0 \\ -1 & ; x<0 \end{cases}$$

∴ f(2) = 1 and f(3) = 1

⇒ f(2) = 1 = f(3)

but 2 ≠ 3

we get same image for different elements

⇒ f is many one

ie, f is not one-one

Again,

2 ∈ codomain such that there exist no element in the domain 'R' for which 2 will becomes image

⇒ f is not onto

also Range (f) = {1,0,-1} ⊂R

⇒ f is not onto

hence, f is neither one-one nor onto fuction.

Chapter-2	Inverse Trigonometric Function प्रतिलोम त्रिकोणमितीय फलन

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 $\sin^{-1}x + \cos^{-1}x =$; $x \in [-1,1]$-

(a) $\pi/4$ (b) $\pi/2$

(c) π (d) $3\pi/2$

Q 2 $\tan^{-1}x + \cot^{-1}x =$; $x \in R$

(a) $3\pi/2$ (b) $\pi/4$

(c) $\pi/2$ (d) π

Q 3 $\operatorname{cosec}^{-1}x + \sec^{-1}x =$; $|x| \ge 1$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) π (d) $\frac{3\pi}{2}$

Q 4 $\tan^{-1}x + \tan^{-1}y =$; $xy < 1$

(a) $\tan^{-1}\frac{x+y}{1-xy}$ (b) $\tan^{-1}\frac{x-y}{1+xy}$

(c) $\tan^{-1}\frac{x+y}{1+xy}$ (d) *None* / कोई नहीं

Q 5 $2\tan^{-1}x =$; $-1 < x < 1$

(a) $\tan^{-1}\frac{2x}{1-x^2}$ (b) $\tan^{-1}\frac{2x}{1+x^2}$

(c) $\tan^{-1}\frac{x}{1-x^2}$ (d) *None* / कोई नहीं

Q 6 $\cos(\sec^{-1}x + \operatorname{cosec}^{-1}x) =$; $|x| \ge 1$

(a) $\frac{\pi}{2}$ (b) 0

(c) π (d) *None* / कोई नहीं

Q 7 $\cot(\tan^{-1}x + \cot^{-1}x) =$?

(a) 1 (b) $\frac{1}{2}$

(c) 0 (d) ∞

Q 8 $\tan^{-1}(1) + \tan^{-1}\left(\frac{1}{3}\right) =$?

(a) $\tan^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{2}{3}\right)$

(c) $\tan^{-1}(2)$ (d) $\tan^{-1}(3)$

Q 9 $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) =$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

Q 10 $\sin\left(\cos^{-1}\frac{3}{5}\right) =$?

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$

(c) $\frac{3}{5}$ (d) *Not possible* / संभव नही हैं।

Q 11 $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) =$?

(a) $8\pi/3$ (b) $\pi/2$

(c) $3\pi/4$ (d) π

Q 12 If $\cot^{-1}\left(-\frac{1}{5}\right) = x$ then $\sin x =$?

यदि $\cot^{-1}\left(-\frac{1}{5}\right) = x$, तो $\sin x$ का मान क्या होगा -

(a) $\frac{1}{\sqrt{26}}$ (b) $\frac{5}{\sqrt{26}}$

(c) $\frac{1}{\sqrt{24}}$ (d) *Not possible* / संभव नही हैं।

Q 13 If $\cot^{-1}(-\sqrt{3}) = x$, where $x \in [0,\pi]$ then value of x is -

यदि $\cot^{-1}(-\sqrt{3}) = x$ जहां $x \in [0,\pi]$ तो x का मान क्या होगा -

(a) $5\pi/6$ (b) $2\pi/3$

(c) $\pi/2$ (d) $\pi/6$

Q 14 **If $\cot^{-1}(-1) = x$, where $x \in [0,\pi]$ then value of x is -**

यदि $\cot^{-1}(-1) = x$ जहां $x \in [0,\pi]$ तो x का मान क्या होगा -

(a) $\pi/2$ (b) $3\pi/4$

(c) π (d) $3\pi/2$

Q 15 $\sin(\tan^{-1}x + \cot^{-1}x) =$; $(x \in R)$

(a) 1 (b) 2

(c) 3 (d) 4

Very Short Question :- (अति लघु उत्तरीय प्रश्न)

2 Marks Question:-

Q 1 Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ in the simplest form .

$\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), x > 1$ को सरलतम रूप मे व्यक्त करें ।

Q 2 Prove that , $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

सिद्ध करें कि $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Q 3 Prove that ,

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3) ; x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

सिद्ध करें कि,

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3) ; x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Q 4 Write in simplest form ,

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right); 0 < x < \pi$$

सरलतम रूप मे व्यक्त करें

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right); 0 < x < \pi$$

Q 5 Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

$\sin^{-1}\left(-\frac{1}{2}\right)$ का मुख्य मान ज्ञात करें ?

Q 6 Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$. का मुख्य मान ज्ञात करें ?

Q 7 If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, then find the value of x .

यदि $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$ तो x का मान ज्ञात करें ।

Q 8 Find the value of , $\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$

$\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right]$ का मान निकाले।

Q 9 If $\tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$, then find the value of x .

यदि $\tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$ तो x का मान ज्ञात करें ।

Short Question : - (लघु उत्तरीय प्रश्न)

3 Marks Question:-

Q 1 Show that , $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

दिखाएँ कि $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Q 2 Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ का मुख्य मान निकाले।

Q 3 Find the value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$ का मान ज्ञात करें।

Q 4 Evaluate , $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

हल करें , $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Q 5. Evaluate , $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

हल करें , $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

Q 6. If $\tan^{-1}\frac{4}{3} = \theta$, then find the value of $\cos\theta$.

यदि $\tan^{-1}\frac{4}{3} = \theta$ तो $\cos\theta$. का मान ज्ञात करें ?

Q 7 Prove that , $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = 3\tan^{-1}\frac{x}{a}$

सिद्ध करें $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = 3\tan^{-1}\frac{x}{a}$

Long Question ; - (दीर्घ उत्तरीय प्रश्न)

5 Marks Question:-

Q 1 Express $\tan^{-1}\frac{\cos x}{1 - \sin x}$; $-\frac{3\pi}{2} < x < \frac{\pi}{2}$ in the simplest form .

सरल रूप में व्यक्त करें,

$\tan^{-1}\frac{\cos x}{1 - \sin x}$; $-\frac{3\pi}{2} < x < \frac{\pi}{2}$

Q 2 Solve $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

हल करें, $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

Q 3 Show that , $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$.

दिखाएं कि, $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$.

Q 4 Evaluate ,

$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

हल करें, $\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

Q 5 Evaluate ,

$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

हल करें, $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

Q 6 Find the value of ,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ का मान निकालें।

Q 7 Prove that,

$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right); x < \pi$

सिद्ध करें, $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right); x < \pi$

Q 8 Prove that, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \left(\frac{\pi}{4} - \frac{x}{2}\right)$

सिद्ध करें, $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \left(\frac{\pi}{4} - \frac{x}{2}\right)$

Q 9 prove that

$$\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

[Hint : Let x= cosθ]

सिद्ध करें, $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

Answer : "Inverse Trigonometric Function Solution"

MCQ 1 - Marks Solution

1 - (b) 2 - (c) 3 - (a) 4 - (a) 5 - (a) 6 - (b) 7 - (c) 8 - (c)
9 - (b) 10 - (b) 11 - (d) 12 - (b) 13 - (a) 14 - (b) 15 - (a)

2- Marks Solution

1 Ans : -

$\because \cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right)$: $x > 1$

let us suppose,

$$x = \sec\theta \quad \Rightarrow \theta = \sec^{-1}x$$

$$\therefore \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{\sec^2\theta - 1}}\right)$$

$$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \cot^{-1}\left(\frac{1}{\tan\theta}\right)$$

$$= \cot^{-1}(\cot\theta)$$

$$= \theta$$

$$\Rightarrow \cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \theta = \sec^{-1}x \quad , \ x > 1$$

2 Ans : L.H.S $= \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$$= \tan^{-1}\left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \times \frac{7}{24}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{48+77}{11 \times 24}}{\frac{264 - 14}{11 \times 24}}\right)$$

$$= \tan^{-1}\left(\frac{125}{250}\right)$$

$$\text{L.H.S} = \tan^{-1}\left(\frac{1}{2}\right) = \text{R.H.S}$$

3 Ans : -

Let us suppose

$$x = \sin\theta \quad \Rightarrow \theta = \sin^{-1}x \quad ...(1)$$

$\because$ L.H.S $= 3\sin^{-1}x$

$$= 3\theta \quad ....(2)$$

and ,

$$\text{R.H.S} = \sin^{-1}(3x - 4x^3)$$
$$= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$$
$$= \sin^{-1}(\sin 3\theta)$$
$$\text{R.H.S} = 3\theta \quad ....(3)$$

From equation (2) and (3) we get,

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$$

4 Ans : -

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) \quad ;0 < x < \pi$$
$$= \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$$
$$= \tan^{-1}\left(\sqrt{\tan^2\frac{x}{2}}\right)$$
$$= \tan^{-1}\left(\tan\frac{x}{2}\right)$$
$$= \frac{x}{2}$$

5 Ans : -

$$\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$$
$$= \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$$
$$= -\frac{\pi}{6}$$

$-\frac{\pi}{6}$ **be the required principal value as** $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

6 Ans : -

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \cos^{-1}\left[-\cos\left(\frac{\pi}{4}\right)\right]$$
$$= \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{4}\right)\right]$$
$$= \pi - \frac{\pi}{4}$$
$$= \frac{3\pi}{4} \in [0, \pi]$$

$\Rightarrow \frac{3\pi}{4}$ **be the required principal value .**

7 Ans : -

$$\because \sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$$
$$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = \sin^{-1}1$$
$$\Rightarrow \cos^{-1}x = \sin^{-1}(1) - \sin^{-1}\left(\frac{1}{5}\right)$$
$$= \sin^{-1}\left(\sin\frac{\pi}{2}\right) - \sin^{-1}\frac{1}{5}$$
$$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}\frac{1}{5}$$
$$\Rightarrow \cos^{-1}x = \cos^{-1}\frac{1}{5}$$
$$\Rightarrow x = \frac{1}{5}$$

8 Ans : -

$$\tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$$

Let $2\tan^{-1}\frac{1}{5} = \theta \quad ....(1)$

$$\Rightarrow \frac{1}{5} = \tan\frac{\theta}{2}$$
$$\Rightarrow \sin\frac{\theta}{2} = \frac{1}{\sqrt{26}} \ \& \cos\frac{\theta}{2} = \frac{5}{\sqrt{26}}$$
$$\therefore \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$$
$$= \tan\left(\theta - \frac{\pi}{4}\right)$$
$$= \frac{\tan\theta - \tan\frac{\pi}{4}}{1 + \tan\theta \times \tan\frac{\pi}{4}}$$
$$= \frac{\tan\theta - 1}{1 + \tan\theta} = \frac{\sin\theta - \cos\theta}{\cos\theta + \sin\theta}$$
$$\Rightarrow \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right)$$
$$= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}$$
$$= \frac{2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} - \left(\frac{25}{26} - \frac{1}{26}\right)}{2 \times \frac{1}{\sqrt{26}} \times \frac{5}{\sqrt{26}} + \left(\frac{25}{26} - \frac{1}{26}\right)}$$
$$= \frac{\frac{5}{13} - \frac{12}{13}}{\frac{12}{13} + \frac{5}{13}} = \frac{-\frac{7}{13}}{\frac{17}{13}}$$
$$\Rightarrow \tan\left(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right) = -\frac{7}{17}$$

9 Ans : -

$$\because \tan^{-1}x + \tan^{-1}3 = \tan^{-1}8$$
$$\Rightarrow \tan^{-1}x = \tan^{-1}8 - \tan^{-1}3$$
$$= \tan^{-1}\left(\frac{8-3}{1+8\times3}\right)$$
$$= \tan^{-1}\left(\frac{5}{1+24}\right) = \tan^{-1}\left(\frac{5}{25}\right)$$
$$\Rightarrow \tan^{-1}x = \tan^{-1}\frac{1}{5}$$
$$\therefore x = \frac{1}{5}$$

3 Marks Solution

1 Ans : -

$$\text{L.H.S} = 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$$
$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$$
$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{7}}{1-\frac{1}{2}\times\frac{1}{7}}\right)$$
$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{9}{14}}{\frac{13}{14}}\right)$$
$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{9}{13}$$
$$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{9}{13}}{1-\frac{1}{2}\times\frac{9}{13}}\right)$$
$$= \tan^{-1}\left(\frac{\frac{31}{26}}{\frac{17}{26}}\right)$$
$$\Rightarrow \text{L.H.S} = \tan^{-1}\left(\frac{31}{17}\right) = \text{R.H.S}$$

2 Ans ; -

$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \sin^{-1}\left(\sin\frac{\pi}{4}\right)$$
$$= \frac{\pi}{4}$$
$$\because \text{Principal interval of sin is } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$
$$\text{and } \frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$
$$\Rightarrow \frac{\pi}{4} \text{ is the principal value of } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

3 Ans : -

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$$
$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(-\sec\frac{\pi}{3}\right)$$
$$= \tan^{-1}\left(\tan\frac{\pi}{3}\right) - \sec^{-1}\left(\sec\left(\pi - \frac{\pi}{3}\right)\right)$$
$$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)$$
$$= \frac{\pi}{3} - \pi + \frac{\pi}{3}$$
$$= \frac{2\pi}{3} - \pi = \frac{2\pi - 3\pi}{3} = -\frac{\pi}{3}$$
$$\Rightarrow \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = -\frac{\pi}{3}$$

4 Ans : -

$$\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$$
$$= \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\sin\frac{\pi}{6}\right)\right)$$
$$= \sin\left[\frac{\pi}{3} - \sin^{-1}\left\{\sin\left(-\frac{\pi}{6}\right)\right\}\right]$$
$$\Rightarrow \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$$
$$= \sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right)$$
$$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$$
$$= \sin\left(\frac{\pi}{2}\right)$$
$$\Rightarrow \sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = 1$$

5 Ans : -

$$\text{Let } \cos^{-1}\left(-\frac{3}{5}\right) = \theta \quad \text{............(1)}$$
$$\Rightarrow \cos\theta = -\frac{3}{5}$$
$$\therefore \sin\theta = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}}$$
$$\Rightarrow \sin\theta = \frac{4}{5}$$
$$\therefore \sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] = \sin(2\theta) \qquad [\text{from (1)}]$$
$$= 2\sin\theta\cdot\cos\theta$$
$$= 2\times\left(\frac{4}{5}\right)\times\left(-\frac{3}{5}\right)$$
$$= -\frac{24}{25}$$
$$\Rightarrow \sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] = -\frac{24}{25}$$

6 Ans : -

$$\because \tan^{-1}\frac{4}{3} = \theta$$

$$\Rightarrow \tan\theta = \frac{4}{3} \text{..............(1)}$$

as $\quad 1 + \tan^2\theta = \sec^2\theta$

$$\Rightarrow 1 + \left(\frac{4}{3}\right)^2 = \sec^2\theta$$

$$\Rightarrow 1 + \frac{16}{9} = \sec^2\theta$$

$$\Rightarrow \frac{25}{9} = \sec^2\theta$$

$$\therefore \sec\theta = \pm\frac{5}{3}$$

$$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\pm\left(5/3\right)}$$

$$\Rightarrow \cos\theta = \pm\frac{3}{5}$$

7 Ans : -

Let $x = a\tan\theta$ ....(1)

$$\therefore \text{L.H.S} = \tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$$

$$= \tan^{-1}\left[\frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right]$$

$$= \tan^{-1}\left[\frac{a^3(3\tan\theta - \tan^3\theta)}{a^3(1 - 3\tan^2\theta)}\right]$$

$$= \tan^{-1}\left[\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right]$$

$$= \tan^{-1}[\tan 3\theta]$$

$$= 3\theta$$

$$\text{L.H.S} = 3\theta = 3\cdot\tan^{-1}\frac{x}{a} = \text{R.H.S}$$

5 Marks Solutions

1 Ans :

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) \quad ; -\frac{3\pi}{2} < x < \frac{\pi}{2}$$

-

$$= \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{1 - 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right)$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} - 2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} + \sin\frac{x}{2}}{\cos\frac{x}{2} - \sin\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1 + \tan\frac{x}{2}}{1 - \tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} + \tan\frac{x}{2}}{1 - \tan\frac{\pi}{4}\cdot\tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right) = \frac{\pi}{4} + \frac{x}{2} \quad ; -\frac{3\pi}{2} < x < \frac{\pi}{2}$$

2 Ans : -

$$\because \tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{2x + 3x}{1 - 2x\cdot 3x}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{5x}{1 - 6x^2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1 - 6x^2} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{1 - 6x^2} = 1$$

$$\Rightarrow 1 - 6x^2 = 5x$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow 6x^2 + 6x - x - 1 = 0$$

$$\Rightarrow 6x(x + 1) - (x + 1) = 0$$

$$\Rightarrow (x + 1)(6x - 1) = 0$$

$\therefore$ either $x + 1 = 0$ or $6x - 1 = 0$

$$\Rightarrow x = -1 \quad \text{or} \quad x = \frac{1}{6}$$

3 Ans : -

let $\sin^{-1}\frac{3}{5} = x$ ....(1)

$$\Rightarrow \sin x = \frac{3}{5}$$

$$\therefore \cos x = \sqrt{1 - \sin^2 x}$$

$= \sqrt{1 - \frac{9}{25}}$

$= \sqrt{\frac{16}{25}}$

$\Rightarrow \cos x = \frac{4}{5}$

and $\sin^{-1}\frac{8}{17} = y$ ....(2)

$\Rightarrow \sin y = \frac{8}{17}$

$\therefore \cos y = \sqrt{1 - \left(\frac{8}{17}\right)^2}$

$= \sqrt{\frac{289 - 64}{289}}$

$= \sqrt{\frac{225}{289}}$

$\Rightarrow \cos y = \frac{15}{17}$

as , $\cos(x - y) = \cos x\cos y + \sin x\sin y$

$= \frac{4}{5} \times \frac{15}{17} + \frac{3}{5} \times \frac{8}{17}$

$= \frac{60}{85} + \frac{24}{85}$

$= \frac{60 + 24}{85}$

$\Rightarrow \cos(x - y) = \frac{84}{85}$

$\Rightarrow x - y = \cos^{-1}\frac{84}{85}$

[by using equation (1) and (2)]

$\Rightarrow \sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$

4 Ans :

$\cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

we know that principal interval of cos is $[0 , \pi]$

$\therefore \cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(-\cos\frac{\pi}{6}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(\cos\left(\pi - \frac{\pi}{6}\right)\right) + \frac{\pi}{6}\right]$

$= \cos\left[\cos^{-1}\left(\cos\frac{5\pi}{6}\right) + \frac{\pi}{6}\right]$ -

$= \cos\left[\frac{5\pi}{6} + \frac{\pi}{6}\right]$; as $\frac{5\pi}{6} \in [0 , \pi]$

$\Rightarrow \therefore \cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right]$

$= \cos\left[\frac{5\pi + \pi}{6}\right]$

$= \cos\left(\frac{6\pi}{6}\right)$

$= \cos(\pi)$

$= -1$

$\therefore \cos\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \frac{\pi}{6}\right] = -1$

5 Ans ; -

$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

we know that principal interval of sin is $\left[-\frac{\pi}{2} , \frac{\pi}{2}\right]$

$\therefore \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

$= \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\sin\frac{\pi}{3}\right)\right]$

$= \sin\left\{\frac{\pi}{2} - \sin^{-1}\left[\sin\left(-\frac{\pi}{3}\right)\right]\right\}$

$= \sin\left[\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right]$

$= \sin\left[\frac{\pi}{2} + \frac{\pi}{3}\right]$

$= \sin\left[\frac{3\pi + 2\pi}{6}\right]$

$= \sin\left[\frac{5\pi}{6}\right]$

$= \cos\frac{\pi}{3}$

$= \frac{1}{2}$

$\therefore \sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \frac{1}{2}$

6 Ans : -

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ ....(1)

we know that principal interval of sin is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, cos is $[0 , \pi]$ and tan is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{2}\right\}$

$\therefore \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$\Rightarrow \tan^{-1} 1 = \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ....(2)

$\cos^{-1}\left(-\frac{1}{2}\right) = \cos^{-1}\left[-\cos\frac{\pi}{3}\right]$

$= \cos^{-1}\left[\cos\left(\pi - \frac{\pi}{3}\right)\right]$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3} \in [0, \pi] \text{ ....(3)}$$

And $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$$= \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$$

$$= -\frac{\pi}{6} \text{ .... (4)}$$

by using eq^{ns} (2),(3) and (4)

$$eq^{n}(1) \Rightarrow \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{3\pi + 8\pi - 2\pi}{12}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

7 Ans : -

$$\text{L.H.S} = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

$$= \tan^{-1}\left[\frac{\cos x(1 - \tan x)}{\cos x(1 + \tan x)}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan x}{1 + \tan x}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \times \tan x}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - x\right)\right]$$

$$= \left(\frac{\pi}{4} - x\right) = \text{R.H.S}$$

hence ,

$$\therefore \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \left(\frac{\pi}{4} - x\right)$$

8 Ans: -

$$\text{L.H.S} = \tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right)$$

$$= \tan^{-1}\left[\frac{\cos 2\frac{x}{2}}{1 + \sin 2\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{1 + 2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right]$$

$$= \tan^{-1}\left[\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} - \tan\frac{x}{2}}{1 + \tan\frac{\pi}{4} \times \tan\frac{x}{2}}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]$$

$$\Rightarrow \text{L.H.S} = \left(\frac{\pi}{4} - \frac{x}{2}\right) = \text{R.H.S}$$

9 Ans :

$$\text{L.H.S} = \tan^{-1}\left\{\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right\}$$

Let $x = \cos\theta$ ....(1)

$$= \tan^{-1}\left\{\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2\cos^2\frac{\theta}{2}} - \sqrt{2\sin^2\frac{\theta}{2}}}{\sqrt{2\cos^2\frac{\theta}{2}} + \sqrt{2\sin^2\frac{\theta}{2}}}\right\}$$

$$= \tan^{-1}\left\{\frac{\sqrt{2}\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)}{\sqrt{2}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{1 - \tan\frac{\theta}{2}}{1 + \tan\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\frac{\tan\frac{\pi}{4} - \tan\frac{\theta}{2}}{1 + \tan\frac{\pi}{4} \times \tan\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right\}$$

$$= \frac{\pi}{4} - \frac{\theta}{2}$$

$$= \frac{\pi}{4} - \frac{1}{2} \times \cos^{-1} x \text{ [by using eqn (1)]}$$

= R.H.S

hence ,

$$\tan^{-1}\left\{\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right\} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

Chapter: 3	Matrices आव्यूह

बहुविकल्पीय प्रश्न (MCQ)

1. यदि $\begin{bmatrix} 1 & 2 \\ x & 3 \end{bmatrix} = \begin{bmatrix} y & z \\ 5 & 3 \end{bmatrix}$ हो तो **x, y** तथा **z** मान होगा–

(a) x=5, y=1, z=2
(b) x=2, y=1, z=5
(c) x=5, y=2, z=1
(d) x=1, y=5, z=2

If $\begin{bmatrix} 1 & 2 \\ x & 3 \end{bmatrix} = \begin{bmatrix} y & z \\ 5 & 3 \end{bmatrix}$ **then the value of x, y and z will be-**

(a) x=5, y=1, z=2
(b) x=2, y=1, z=5
(c) x=5, y=2, z=1
(d) x=1, y=5, z=2

2. A and B are two matrices then $(A+B)^2$ is equal to -

(a) $A^2 + 2AB + B^2$
(b) $A^2 + AB + BA + B^2$
(c) $A^2 + 2AB + 2BA + B^2$
(d) Not possible.

यदि A और B दो आव्यूह हो तो $(A+B)^2$ का मान होगा–

(a) $A^2 + 2AB + B^2$
(b) $A^2 + AB + BA + B^2$
(c) $A^2 + 2AB + 2BA + B^2$
(d) संभव नहीं हैं।

3. If A and B are two matrices then (AB)' is equal to -

(a) A'B'
(b) B'A'
(c) (BA)'
(d) Not defined.

यदि A और B दो आव्यूह हो तो (AB)' का मान होगा–

(a) A'B'
(b) B'A'
(c) (BA)'
(d) परिभाषित नहीं ।

4. If A=$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ then which of the following is A'-

(a) $\begin{bmatrix} 2 & 2 & 3 \\ 5 & 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 \\ 2 & 5 \\ 1 & 4 \end{bmatrix}$

यदि A=$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ हो तो निम्नांकित में कौन A' के समान है–

(a) $\begin{bmatrix} 2 & 2 & 3 \\ 5 & 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 6 \\ 2 & 5 \\ 1 & 4 \end{bmatrix}$

5. If A and B are two matrices then (A+B)' is equal to -

यदि A एंव B दो आव्यूह हो तो (A+B)' के बराबर होगा–

(a) A'+B'
(b) B'+A'
(c) (AB)'
(d) Option (a) and (b) / विकल्प (a) और (b)

6. A square matrix B is said to be a symmetric Matrix if -

एक वर्ग आव्यूह B समामित आव्यूह होगा यदि –

(a) B=B'
(b) B=-B'
(c) B=-B
(d) None of these / कोई नहीं

7. A square matrix B is said to be a skew-symmetric matrix if -

एक वर्ग आव्यूह B विषम सममित आव्यूह होगा यदि–

(a) B=B'
(b) B=-B'
(c) B=-B
(d) None of these / कोई नहीं

8. **A square matrix B is said to be a singular matrix if-**

एक वर्ग आव्यूह B अव्युत्क्रमणीय आव्यूह होगा यदि–

(a) $|B| = 0$ (b) $|B| = 1$

(c) $|B| = 2$ (d) $|B| = 3$

9. **A square matrix B is said to be a non singular matrix if-**

एक वर्ग आव्यह B व्युत्क्रमणीय आव्यूह होगा यदि–

(a) $|B| = 0$ (b) $|B| \neq 0$

(c) $|B| = 1$ (d) $|B| = 2$

10. **If A be a matrix then AA^{-1} is equal to-**

यदि A एक आव्यूह हो तो AA^{-1} का मान होगा–

(a) I / unit matrix / **एकांक आव्यूह**

(b) $A^{-1}A$

(c) $1/2$

(d) Not defined / **परिभाषित नहीं।**

11. **If A and B are two matrices then $(AB)^{-1}$ is-**

यदि A और B दो आव्यूह हो तो $(AB)^{-1}$ होगा–

(a) $B^{-1}A^{-1}$

(b) $A^{-1}B^{-1}$

(c) $(BA)^{-1}$

(d) Undefined / अपरिभाषित

12. **If A be a square matrix then A^{-1} will exist if-**

यदि A एक वर्ग आव्यूह हो तो A^{-1} संभव होगा यदि–

(a) A is a singular / अव्युत्क्रमणीय

(b) A is non-singular / व्युत्क्रमणीय

(c) Row-matrix / पंक्ति आव्यूह

(d) Column matrix / स्तम्भ आव्यूह

13. **Which of the following is a non-singular matrix?**

निम्नलिखित में कौन व्युत्क्रमणीय आव्यूह हैं ?

(a) $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & -6 \\ 6 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -6 \\ -6 & 9 \end{bmatrix}$ **(d)** $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}$

14. **Which of the following is a singular matrix ?**

निम्नलिखित में कौन अव्युत्क्रमणीय आव्यूह हैं ?

(a) $\begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -6 \\ 6 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

15. **For which of the following the inverse matrix can be obtained ?**

निम्नलिखित में किसका व्युत्क्रम ज्ञात किया जा सकता हैं?

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **(d)** $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

16. **A matrix A= $[a_{ij}]_{n\times n}$ is said to be symmetric if -**

एक आव्यूह A= $[a_{ij}]_{n\times n}$ सममित आव्यूह होगा यदि–

(a) $a_{ij} = 0$

(b) $a_{ij} = -a_{ji}$

(c) $a_{ij} = a_{ji}$

(d) $a_{ij} = 1$

17. **A matrix A=$[a_{ij}]_{n\times n}$ is said to be a skew-symmetric if -**

एक आव्यूह A=$[a_{ij}]_{n\times n}$ विषम सममित आव्यूह होगा यदि –

(a) $a_{ij} = 0$

(b) $a_{ij} = -a_{ji}$ and $a_{ii} = 0$

(c) $a_{ij} = a_{ji}$

(d) $a_{ij} = 1$

MATRICES (2 MARKS QUESTIION)

1. **For which values of x and y the given metrices are equal-**

$\begin{bmatrix} 0 & x-2 \\ 8 & 4 \end{bmatrix}$ **and** $\begin{bmatrix} 3y+7 & 5 \\ x+1 & 2-3y \end{bmatrix}$

x और y के कौन से मान के लिए दिया गया आव्यूह समान होंगे–

$\begin{bmatrix} 0 & x-2 \\ 8 & 4 \end{bmatrix}$ और $\begin{bmatrix} 3y+7 & 5 \\ x+1 & 2-3y \end{bmatrix}$

2.Find the value of x from the following -

निम्न से x का मान ज्ञात कीजिए –

$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$

3. If A $=\begin{bmatrix}2 & 3 & 1\\ 0 & -1 & 5\end{bmatrix}$ and B= $\begin{bmatrix}1 & 2 & -1\\ 0 & -1 & 3\end{bmatrix}$ then find the matrix (2A - 3B)

यदि A= $\begin{bmatrix}2 & 3 & 1\\ 0 & -1 & 5\end{bmatrix}$ और B= $\begin{bmatrix}1 & 2 & -1\\ 0 & -1 & 3\end{bmatrix}$ तो (2A - 3B) को ज्ञात करे –

4. Evaluate / मान ज्ञात करें–

$$3\begin{bmatrix}1 & 3 & 4\\ 2 & 5 & 6\end{bmatrix}-2\begin{bmatrix}2 & 5 & 7\\ 3 & 4 & 8\end{bmatrix}+4\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & -3\end{bmatrix}$$

5. Construct a 2x2 matrix whoes elements are given by $a_{ij}=(2_i-j)$

एक 2x2 आव्यूह की रचना करें जहाँ $a_{ij}=(2_i-j)$ हैं।

6. Construct a 3x3 matrix where $a_{ij}={}^{i}\!/_{j}$

एक 3x3 आव्यूह की रचना करें जहाँ $a_{ij}={}^{i}\!/_{j}$ हो.

7. Give an example of 3x3 matrix which is a zero matrix ?

एक 3x3 शून्य आव्यूह का उदाहरण दें ?

8. If A= $\begin{bmatrix}2 & 3 & 5\\ -1 & 0 & 4\end{bmatrix}$ and B= $\begin{bmatrix}4 & -2 & 3\\ 2 & 6 & -1\end{bmatrix}$ then verify that A+B=B+A ?

यदि A= $\begin{bmatrix}2 & 3 & 5\\ -1 & 0 & 4\end{bmatrix}$ और B= $\begin{bmatrix}4 & -2 & 3\\ 2 & 6 & -1\end{bmatrix}$ तो सिद्ध करें कि A+B=B+A ?

9. Find a matrix X such that 2A+B+X=0, where A= $\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$ and B= $\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$?

आव्यूह X का मान ज्ञात करें यदि 2A+B+X=0 जहाँ A= $\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}$ और B= $\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$?

MATRICES (3 Marks Question)

1. If $\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 30 & 3 & i\end{vmatrix}=x+iy$ then find the value of x and y?

यदि $\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 30 & 3 & i\end{vmatrix}=x+iy$ हो तो x और y का मान ज्ञात करें ?

2. By using elementary row operation find the inverse of the matrix-

प्रारंभिक पंक्ति संक्रियाओं का प्रयोग करते हुए आव्यूह का व्युत्क्रम ज्ञात करें–

$$A=\begin{bmatrix}3 & -1\\ -4 & 2\end{bmatrix}$$

3. If A = $\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$ and B = $\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 1 & 3 & 2\end{bmatrix}$ then (AB)=

यदि A = $\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$ और B = $\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 1 & 3 & 2\end{bmatrix}$ तो (AB)=

4. If A= $\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ then show that $A^3=4A$?

यदि A= $\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ हो तो दिखाएँ कि $A^3=4A$?

5. Find the value of / मान निकालें–

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}=$$

MATRICES (5 Markes question)

1. If A= $\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$ then find the value of k such that $A^2=8A+kI$

यदि A= $\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$ तो k का वह मान निकाले जिसके लिए $A^2=8A+kI$ हो।

2. If A = $\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$ then show that

$A^2-5A-14I=0$?

यदि A = $\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$ हो तो दर्शाएँ कि

$A^2-5A-14I=0$?

3. If A = $\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then prove that -

$$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix};\ n\in N.$$

यदि $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ तो सिद्ध करे कि

$A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$; $n\in N$.

4. If $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}=0$ then find the value of x?

यदि $\begin{bmatrix}1 & x & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}=0$, हो तो x का मान ज्ञात करें ?

5. If $A=\begin{bmatrix}-3\\ 5\\ 2\end{bmatrix}$ and $B=\begin{bmatrix}1 & 6 & -4\end{bmatrix}$ then verify that (AB)'=B'A' ?

यदि $A=\begin{bmatrix}-3\\ 5\\ 2\end{bmatrix}$ और $B=\begin{bmatrix}1 & 6 & -4\end{bmatrix}$ हो तो दर्शाएँ कि (AB)'=B'A' ?

6. If matrix $A=\begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$ then find the corrresponding symmetric part and skew-symmetric part of the matrix?

यदि $A=\begin{bmatrix}1 & 3 & 5\\ -6 & 8 & 3\\ -4 & 6 & 5\end{bmatrix}$ हो तो A के संगत सममित आव्यूह तथा विसम सममित आव्यूह को लिखें?

7. By using elementary row operation find the inverse of the matrix

$$A=\begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}$$

प्रारंभिक पंक्ति संक्रियाओं का प्रयोग करते हुए आव्यूह का व्युत्क्रम ज्ञात करें –

$$A=\begin{bmatrix}2 & -3 & 3\\ 2 & 2 & 3\\ 3 & -2 & 2\end{bmatrix}$$

8. If $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ then prove that

$|\text{adj } A|=|A|^2$

यदि $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$ तो सिद्ध करें कि

$|\text{adj } A|=|A|^2$

9. If $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ then find the matrices (A + A') and (A - A'). Also prove that (A+A') is a symmetric matrix while (A - A') is a skew-symmetric matrix ?

यदि $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$ तो आव्यूह (A+A') और (A-A') ज्ञात करें ।

पुनः सिद्ध करें कि (A+A') सममित आव्यूह है किन्तु (A-A') एक विषम सममित आव्यूह है ।

10. If $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ then prove that f(x+y)=f(x).f(y).

यदि $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$ तो सिद्ध करें कि f(x+y)=f(x).f(y).

MATRICES (SOLUTION)

MCQ:-

1) (a)
2) (b)
3) (b)
4) (c)
5) (d)
6) (a)
7) (b)
8) (a)
9) (b)
10) (a)
11) (a)
12) (b)
13) (b)
14) (a)
15) (c)
16) (c)
17) (b)

2 Marks Solution

1) $$\begin{bmatrix} 0 & x-2 \\ 8 & 4 \end{bmatrix} = \begin{bmatrix} 3y+7 & 5 \\ x+1 & 2-3y \end{bmatrix}$$

$\because$ two matrices are equal if corresponding position elements are same.

$\therefore$ 3y+7=0 -------(1)

x-2=5 -------(2)

x+1=8 -------(3)

and 2-3y=4 ---(4)

eq^n-(1) $\Rightarrow$ y= $\frac{-7}{3}$

eq^n-(2) $\Rightarrow$ x=7

eq^n-(3) $\Rightarrow$ x=7

eq^n-(4) $\Rightarrow$ y= $\frac{-2}{3}$

$\because$ y= $\frac{-7}{3} \neq \frac{-2}{3}$

hence for no value of x and y given matrices are equal

(2). $$\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$$

$\because$ 2x-y = 6 ------(1)

and y= -2 -----(2)

$\therefore$ (1)$\Rightarrow$ x = 2 and y = - 2

(3) $$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\therefore (2A-3B) = 2\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 2 \\ 0 & -2 & 10 \end{bmatrix} - \begin{bmatrix} 3 & 6 & -3 \\ 0 & -3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 6-6 & 2+3 \\ 0-0 & -2+3 & 10-9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix}$$

(4) $$3\begin{bmatrix} 1 & 3 & 4 \\ 2 & 5 & 6 \end{bmatrix} - 2\begin{bmatrix} 2 & 5 & 7 \\ 3 & 4 & 8 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 9 & 12 \\ 6 & 15 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 10 & 14 \\ 6 & 8 & 16 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 8 \\ 8 & 4 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4+4 & 9-10+0 & 12-14+8 \\ 6-6+8 & 15-8+4 & 18-16-12 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 & 6 \\ 8 & 11 & -10 \end{bmatrix}$$

(5)

$\because \ a_{ij} = (2i-j)$

$$\therefore \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times2} = \begin{bmatrix} 2\times1-1 & 2\times1-2 \\ 2\times2-1 & 2\times2-2 \end{bmatrix}_{2\times2}$$

$$= \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}_{2\times2}$$

(6) $\because \ a_{ij} = i/j$

$$\therefore \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1/1 & 1/2 & 1/3 \\ 2/1 & 2/2 & 2/3 \\ 3/1 & 3/2 & 3/1 \end{bmatrix}_{3X3}$$

$$= \begin{bmatrix} 1 & 1/2 & 1/3 \\ 2 & 1 & 2/3 \\ 3 & 3/2 & 1 \end{bmatrix}_{3X3}$$

(7)

$$0_{3X3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3X3}$$

(8)

$$A = \begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix}$$

$$\therefore A+B = \begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4 & 3-2 & 5+3 \\ -1+2 & 0+6 & 4-1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & 8 \\ 1 & 6 & 3 \end{bmatrix} \text{---------- (1)}$$

and $$B+A = \begin{bmatrix} 4 & -2 & 3 \\ 2 & 6 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 5 \\ -1 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & -2+3 & 3+5 \\ 2-1 & 6+0 & -1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & 8 \\ 1 & 6 & 3 \end{bmatrix} \text{---------- (2)}$$

from eqn (1) and (2)

A+B=B+A

(9)

$$A=\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix} \text{ and } B=\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}$$

$$\because 2A+B+X=0$$

$$\therefore X=-(2A+B)$$

$$=-\left\{2\begin{bmatrix}-1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}\right\}$$

$$=-\left\{\begin{bmatrix}-2 & 4\\ 6 & 8\end{bmatrix}+\begin{bmatrix}3 & -2\\ 1 & 5\end{bmatrix}\right\}$$

$$=-\begin{bmatrix}-2+3 & 4-2\\ 6+1 & 8+5\end{bmatrix}$$

$$=-\begin{bmatrix}1 & 2\\ 7 & 13\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -2\\ -7 & -13\end{bmatrix}$$

3 Marks solution :-

(1)

$$\begin{vmatrix}6i & -3i & 1\\ 4 & 3i & -1\\ 30 & 3 & i\end{vmatrix}=x+iy$$

$$\Rightarrow 6i(-3+3)+3i(4i+30)+(12-90i)=x+iy$$

$$\Rightarrow 0-12+90i+12-90i=x+iy$$

$$\Rightarrow 0=x+iy$$

$$\Rightarrow x=0 \text{ and } y=0$$

(2)

$$A=\begin{bmatrix}3 & -1\\ -4 & 2\end{bmatrix}$$

as, A = IA

$$\Rightarrow\begin{bmatrix}3 & -1\\ -4 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.A$$

$$R1\to\frac{R1}{3}$$

$$\Rightarrow\begin{bmatrix}1 & -1/3\\ -4 & 2\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 0 & 1\end{bmatrix}.A$$

$$R2\to R2+4R1$$

$$\Rightarrow\begin{bmatrix}1 & -1/3\\ 0 & 2-4/3\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 4/3 & 1\end{bmatrix}.A$$

$$\begin{bmatrix}1 & -1/3\\ 0 & 2/3\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 4/3 & 1\end{bmatrix}.A$$

$$R2\to\frac{3}{2}R2$$

$$\Rightarrow\begin{bmatrix}1 & -1/3\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1/3 & 0\\ 2 & 3/2\end{bmatrix}.A$$

$$R1\to R1+\frac{1}{3}R2$$

$$\Rightarrow\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\frac{1}{3}+\frac{2}{3} & \frac{1}{2}\\ 2 & \frac{3}{2}\end{bmatrix}.A$$

i.e $I_2=A^{-1}.A$

$$\Rightarrow A^{-1}=\begin{bmatrix}1 & 1/2\\ 2 & 3/2\end{bmatrix}_{2X2}$$

(3)

$$\because A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 1 & 3 & 2\end{bmatrix}$$

$$\therefore AB=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}.\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\\ 1 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}1+0+0 & 2+0+0 & 3+0+0\\ 0+6+0 & 0+4+0 & 0+2+0\\ 0+0+3 & 0+0+9 & 0+0+6\end{bmatrix}$$

$$=\begin{bmatrix}1 & 2 & 3\\ 6 & 4 & 2\\ 3 & 9 & 6\end{bmatrix}$$

(4)

$$\because A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}.\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$A^2=\begin{bmatrix}1+1 & -1-1\\ -1-1 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}$$

$$A^3=A^2.A$$

$$=\begin{bmatrix}2 & -2\\ -2 & 2\end{bmatrix}.\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & -2-2\\ -2-2 & 2+2\end{bmatrix}$$

$$=\begin{bmatrix}4 & -4\\ -4 & 4\end{bmatrix}$$

$$=4\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow A^3=4A$$

(5)

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ -\sin\theta\cos & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I_2$$

5 Marks solution

(1)

$\because A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$

$A^2 = A.A$

$=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}.\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$

$=\begin{bmatrix}1+0 & 0+0\\ -1-7 & 0+49\end{bmatrix}$

$A^2=\begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix}$

as, $A^2 = 8A + kI$

$\Rightarrow \begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix} = 8\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix} + k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}8 & 0\\ -8 & 56\end{bmatrix} + k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\begin{bmatrix}1 & 0\\ -8 & 49\end{bmatrix} - \begin{bmatrix}8 & 0\\ -8 & 56\end{bmatrix} = k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\begin{bmatrix}1-8 & 0-0\\ -8+8 & 49-56\end{bmatrix} = k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix} = k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$(-7)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = k\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

hence, $k=-7$

(2)

$A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$

$\therefore A^2 = A.A = \begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}.\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$

$=\begin{bmatrix}9+20 & -15-10\\ -12-8 & 20+4\end{bmatrix}$

$A^2=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}$

Now, L.H.S = $A^2 - 5A - 14I$

$=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix} - 5\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix} - 14\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix} - \begin{bmatrix}15 & -25\\ -20 & 10\end{bmatrix} - \begin{bmatrix}14 & 0\\ 0 & 14\end{bmatrix}$

$=\begin{bmatrix}29-15-14 & -25+25-0\\ -20+20-0 & 24-10-14\end{bmatrix}$

$\Rightarrow$ L.H.S $=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ = R.H.S Ans..

(3)

$\because A = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\Rightarrow A^2 = A.A$

$=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}.\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta - \sin^2\theta & \sin\theta\cos\theta + \sin\theta\cos\theta\\ -\sin\theta\cos\theta - \sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos 2\theta & 2\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & \cos 2\theta\end{bmatrix}$

$A^2 = \begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

$\therefore$ Again ,

$A^3 = A^2.A$

$=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}.\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos\theta\cos 2\theta - \sin\theta\sin 2\theta & \sin\theta\cos 2\theta + \cos\theta\sin 2\theta\\ -\cos\theta\sin 2\theta - \sin\theta\cos 2\theta & -\sin\theta\sin 2\theta + \cos\theta\cos 2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos(\theta+2\theta) & \sin(\theta+2\theta)\\ -\sin(\theta+2\theta) & \cos(\theta+2\theta)\end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix}\cos 3\theta & \sin 3\theta\\ -\sin 3\theta & \cos 3\theta\end{bmatrix}$

similarly ,

$A^n = \begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}; n\in N$

(4)

$[1 \;\; x \;\; 1]_{1\times 3}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 3 & 2 & 5\end{bmatrix}_{3\times 3}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}_{3\times 1} = 0$

$\Rightarrow [1+4x+3 \;\; 2+5x+2 \;\; 3+6x+5]_{1\times 3}\begin{bmatrix}1\\ -2\\ 3\end{bmatrix}_{3\times 1} = 0$

$\Rightarrow [(1+4x+3) - 2(2+5x+2) + 3(3+6x+5)] = 0$

$\Rightarrow (4x+4) - 2(5x+4) + 3(6x+8) = 0$

$\Rightarrow 4x + 4 - 10x - 8 + 18x + 24 = 0$

$\Rightarrow (4x - 10x + 18x) + (4 - 8 + 24) = 0$

$\Rightarrow 12x + 20 = 0$

$\Rightarrow \therefore x = -\frac{20}{12} = -\frac{5}{3}$

(5)

$$\because A = \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 6 & -4 \end{bmatrix}$$

$$\therefore A.B = \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}_{3\times 1} . \begin{bmatrix} 1 & 6 & -4 \end{bmatrix}_{1\times 3}$$

$$\Rightarrow A.B = \begin{bmatrix} -3 & -18 & 12 \\ 5 & 30 & -20 \\ 2 & 12 & -8 \end{bmatrix}_{3\times 3}$$

$$\therefore (AB)' = \begin{bmatrix} -3 & 5 & 2 \\ -18 & 30 & 12 \\ 12 & -20 & -8 \end{bmatrix} \text{--- (i)}$$

Now,

$$B'A' = \begin{bmatrix} 1 \\ 6 \\ -4 \end{bmatrix}_{3\times 1} \begin{bmatrix} -3 & 5 & 2 \end{bmatrix}_{1\times 3}$$

$$= \begin{bmatrix} -3 & 5 & 2 \\ -18 & 30 & 12 \\ 12 & -20 & -8 \end{bmatrix}_{3\times 3} \text{--- (ii)}$$

from equqtion (i) and (ii)

$(AB)' = B'A'$

(6)

$$\because A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}_{3\times 3}$$

$$A' = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}_{3\times 3}$$

Hence corresponding symmetric part

$$= \frac{1}{2}(A + A')$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 1+1 & 3-6 & 5-4 \\ -6+3 & 8+8 & 3+6 \\ -4+5 & 6+3 & 5+4 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 9 \end{bmatrix}$$

Agian

$$\text{Skew - symmetric part} = \frac{1}{2}(A - A')$$

$$= \frac{1}{2}\left\{\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} - \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} \cancel{1}-\cancel{1} & 3+6 & 5+4 \\ -6-3 & \cancel{8}-\cancel{8} & 3-6 \\ -4-5 & 6-3 & \cancel{5}-\cancel{5} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

(7)

$$\because A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$$

as A = IA

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. A$$

$R_1 \leftrightarrows R_3$

$$\Rightarrow \begin{bmatrix} 3 & -2 & 2 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}. A$$

$R_1 \to R_1 - R_3,\ R_2 \to R_2 - R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 5 & 0 \\ 2 & -3 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}. A$$

$R_3 \to R_3 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 5 & 0 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix}. A$$

$R_2 \to \frac{R_2}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -1/5 & 1/5 & 0 \\ 3 & 0 & -2 \end{bmatrix}. A$$

$R_1 \to R_1 - R_2,\ R_3 \to R_3 + 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -4/5 & -1/5 & 1 \\ -1/5 & 1/5 & 0 \\ 2 & 1 & -2 \end{bmatrix}. A$$

$R_3 \to \frac{R_3}{5}$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4/5 & -1/5 & 1 \\ -1/5 & 1/5 & 0 \\ 2/5 & 1/5 & -2/5 \end{bmatrix}. A$$

$R_1 \to R_1 + R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/5 & 0 & 3/5 \\ -1/5 & 1/5 & 0 \\ 2/5 & 1/5 & -2/5 \end{bmatrix}. A$$

$$\textit{Hence, } A^{-1} = \begin{bmatrix} -2/5 & 0 & 3/5 \\ -1/5 & 1/5 & 0 \\ 2/5 & 1/5 & -2/5 \end{bmatrix}$$

(8)

$\because\ A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$

$= 1(8-6)+(0+9)+2(0-6)$

$= 2+9-12$

$|A| = -1$ ---- (i)

Corresponding, minors of matrix 'A' is -

$a_{11} = \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = 8-6 = 2$

$a_{12} = \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = 0+9 = 9$

$a_{13} = \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = 0-6 = -6$

$a_{21} = \begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = -4+4 = 0$

$a_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4-6 = -2$

$a_{23} = \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -2+3 = 1$

$a_{31} = \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3-4 = -1$

$a_{32} = \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -3-0 = -3$

$a_{33} = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2-0 = 2$

$\therefore$ cofactor matrix $= \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}$

$\therefore$ adj(A) = [cofactor matrix]' $= \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

$\Rightarrow |adj(A)| = 2(-4+3)-0-(9-12)$

$= 2\times(-1)-(-3)$

$= -2+3 = 1$

$= (-1)^2$

$\Rightarrow |adj(A)| = |A|^2$ [by uisng eqn (i)]

(9)

$\because\ A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 1-1 & -1+1 \\ -1+1 & 0+0 & 1-1 \\ 1-1 & -1+1 & 0+0 \end{bmatrix}$

$\Rightarrow (A+A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ --- (i)

and $A - A' = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0-0 & 1+1 & -1-1 \\ -1-1 & 0-0 & 1+1 \\ 1+1 & -1-1 & 0-0 \end{bmatrix}$

$\Rightarrow (A-A') = \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix}$ ---- (ii)

Again,

$\because (A+A')' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow (A+A')' = (A+A')$

$\Rightarrow (A+A')$ is a symmetric matrix

Also,

$(A-A')' = \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -2 & 2 \\ 2 & 0 & -2 \\ -2 & 2 & 0 \end{bmatrix}$

$= -\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 2 \\ 2 & -2 & 0 \end{bmatrix}$

$\Rightarrow (A-A')' = -(A-A')$

$\Rightarrow (A-A')$ is a skew - symmetric matrix.

10).

$$\because F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x+y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix} \text{---- (i)}$$

$$f(x+y)=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\sin x\cos y-\cos x\sin y & 0\\ \sin x\cos y+\cos x\sin y & \cos x\cos y-\sin x\sin y & 0\\ 0 & 0 & 1\end{bmatrix}$$

Again,

$$f(x).f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}.\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x\sin y & -\sin y\cos x-\sin x\cos y & 0\\ \sin x\cos y+\cos x\sin y & -\sin x\sin y+\cos x\cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$f(x).f(y)=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix} \text{---- (ii)}$$

from eqn (i) and (ii) we get ,

$f(x+y)=f(x).f(y)$

Chapter - 4 Determinants
सारणिक

Multiple choice Questions
बहुविकल्पीय प्रश्न

Q.1. IF $A = \begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$ **then** $|A|$ **?**

यदि $\begin{bmatrix} 2 & 4 \\ -5 & -1 \end{bmatrix}$ तो $|A| = ?$

(a) -18 (b) 18
(c) 0 (d) 12

Q.2. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}$ **then x is equal to**

यदि $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}$ तो x का मान है –

(a) 6 (b) ± 6
(c) -6 (d) zero

Q.3. $\begin{vmatrix} \sin 10^o & -\cos 10^o \\ \sin 80^o & \cos 80^o \end{vmatrix} =$

(a) 1 (b) -1
(c) ± 1 (d) 0

Q.4. If A is a square Matrix then $|A'| = ?$

यदि A एक वर्ग आव्यूह है तो $|A'| = ?$

(a) A (b) $-|A|$
(c) $|A|$ (d) A'

Q.5. Let A be a square Matrix of order 3 × 3 and $|A| = 2$ **then** $|2A|$ **is euqal to**

माना A एक वर्ग आव्यूह है 3 × 3 कोटि का और $|A| = 2$ तो $|2A|$ का मान होगा ?

(a) 16 (b) -16
(c) 8 (d) -8

Q.6. If I is an Identity Matrix then $|I| = ?$

यदि I एक तत्समक आव्यूह है तो $|I| = ?$

(a) -1 (b) -2
(c) 1 (d) 2

Q.7. If $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix}$ **= 0 then value of x is**

यदि $\begin{vmatrix} x & 2 \\ 2 & 1 \end{vmatrix} = 0$ तो x का मान है

(a) 2 (b) -4
(c) 5 (d) 1

Q.8. $\begin{vmatrix} 2 & 3 & 4 \\ 0 & 5 & 6 \\ 0 & 0 & 2 \end{vmatrix} = ?$

(a) 20 (b) 30
(c) 40 (d) -20

Q.9. Determinant of null square matrix will be

शुन्य वर्ग आव्यूह का सारणिक मान होगा?

(a) 0 (b) 1
(c) 6 (d) None

Q.10. Area of triangle is always

त्रिभुज का क्षेत्रफल हमेशा होता है –

(a) Negative ऋणात्मक (b) Positive धनात्मक
(c) Zero शून्य (d) None of these इनमें से कोई नहीं

Q.11. A square Matrix A is invertible if

एक वर्ग आव्यूह व्युत्क्रमणीय है, यदि –

(a) $|A| = 0$ (b) $|A| = 1$
(c) $|A| \neq 0$ (d) None of these

Q.12. If A is invertible matrix then A^{-1} **will be**

यदि A व्युत्क्रमणीय आव्यूह है तो A^{-1} का मान होगा

(a) adj A (b) $\frac{1}{|A|}$ adj A
(c) $\frac{-1}{|A|}$adj A (d) None of these

Q.13. $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 8 \end{vmatrix} = ?$

(a) 8 (b) 40
(c) 48 (d) 50

Q.14. $\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = ?$

(a) - 4 (b) 2
(c) 6 (d) 0

Q.15. $\begin{vmatrix} 0 & 0 \\ 5 & 8 \end{vmatrix} = ?$

(a) 5 (b) 40
(c) 8 (d) 0

Q.16. Which of the following is correct?

निम्नलिखित में से कौन सही है?

(a) $|A| = |A'|$ (b) $|A| = -|A'|$
(c) $|A| \neq |A'|$ (d) None

Q.17. Value of the determinant of a matrix exist , if matrix is -

किसी आव्यूह के सारणिक मान संभव है यदि आव्यूह है :–

(a) square matrix (वर्ग आव्यूह) (b) Rectangle Matrix (आयत आव्यूह)
(c) Any Matrix कोई भी आव्यूह (d) None of these इनमें से कोई नहीं

Q.18. If $\begin{vmatrix} x & 2 \\ 3 & 1 \end{vmatrix} = 8$ **then value of x will be**

यदि $\begin{vmatrix} x & 2 \\ 3 & 1 \end{vmatrix} = 8$ **तो x का मान होगा**

(a) 8 (b) 6
(c) 14 (d) 0

Q.19. $\begin{vmatrix} 0 & 2 & 4 \\ 0 & 0 & -7 \\ 0 & 0 & 0 \end{vmatrix} = ?$

(a) 0 (b) 8
(c) -14 (d) -56

Q.20. $\begin{vmatrix} 2 & 3 & 1 \\ 5 & 6 & 8 \\ 2 & 3 & 1 \end{vmatrix} = ?$

(a) 5 (b) -1
(c) 0 (d) None of these

Q.21. $\begin{vmatrix} 7 & 8 & -1 \\ 5 & 4 & 3 \\ 5 & 4 & 3 \end{vmatrix} = ?$

(a) 3 (b) 1
(c) 0 (d) 2

Q.22. $\begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = ?$

(a) 0 (b) 1
(c) 2 (d) 3

Q.23. $\begin{vmatrix} 5 & 4 \\ 2 & 8 \end{vmatrix} = ?$

(a) 40 (b) 48
(c) 42 (d) 32

Q.24. $\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = ?$

(a) 2 (b) 1
(c) 4 (d) 0

Q.25. If $\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$ **then co-factor of 5 is**

यदि $\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$ **तो 5 का सहखण्ड है**

(a) 6 (b) -6
(c) 2 (d) 4

Q.26. Minor of a_{21} in $\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix}$ **is**

$\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix}$ **में a_{21} का Minor है**

(a) 5 (b) 2
(c) 6 (d) 0

Q 27. If $\begin{vmatrix} 3 & 4 & 7 \\ 2 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$ **then** $M_{11} = ?$

(a) 3 (b) 1
(c) 2 (d) -1

Q 28. If $A = \begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}$ **then adj A = ?**

(a) $\begin{bmatrix} 2 & 3 \\ -5 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 4 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & 4 \\ 2 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix}$

Q 29. If $A_{n \times n}$ Matrix then $|kA| = ?$, k is constant-

यदि $A_{n \times n}$ एक आव्यूह है तो $|kA| = ?$, k नियत है।

(a) $k|A|$ (b) $k^n|A|$

(c) $k.|A|^n$ (d) $k^n|A|^n$

Q 30. Determinant of diagonal Matrix diag. [5, 6, 7] is-

विकर्ण आव्यूह diag [5, 6, 7] का सारणिक मान है:

(a) 30 (b) 42

(c) 35 (d) 210

MCQ Ans:-
1-b, 2-b, 3-a, 4-c, 5-a, 6-c, 7-b, 8-a, 9-a, 10-b, 11-c, 12-b, 13-c, 14-a, 15-d, 16-a, 17-a, 18-c, 19-a, 20-c, 21-c, 22-a, 23-d, 24-d, 25-a, 26-c, 27-d, 28-d, 29-b, 30-d

Very Short Questions (2 Marks)
अतिलघुत्तरीय प्रश्न

Q.1. Evaluate

मान ज्ञात करें।

(a) $\begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \\ 5 & 0 & 3 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 3 \end{vmatrix}$ (d) $\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{vmatrix}$

(e) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$

Q.2. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, then find the value of x.

यदि $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$ तो x का मान ज्ञात करें।

Q.3. Write the co-factor of all elements of $\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}$

सारणिक $\begin{vmatrix} 3 & -2 \\ 4 & 6 \end{vmatrix}$ के सभी अवयवों के सहखण्ड लिखें।

Q.4. Prove that $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$

Where, $1 + w + w^2 = 0$

सिद्ध करें $\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$

जहाँ, $1 + w + w^2 = 0$

Q.5. If $\begin{vmatrix} x-9 & 1 \\ 7 & 1 \end{vmatrix} = 0$, then find the value of x.

यदि $\begin{vmatrix} x-9 & 1 \\ 7 & 1 \end{vmatrix} = 0$ तो x का मान ज्ञात करें।

Q.6. Write the Matrix form of the equations

$2x - y = -2, 3x + 4y = 3.$

समीकरण $2x - y = -2, 3x + 4y = 3$ को आव्यूह रूप में लिखें।

Q.7. Evaluate $\begin{vmatrix} 5 & 60 & 7 \\ 3 & 36 & 2 \\ 2 & 24 & 4 \end{vmatrix}$

Solutions of Very Short Questions (2 Marks)

1. (a) $\begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} = 5 \times 6 - 2 \times 4$

$= 30 - 8 = 22$ Ans

(b) $\begin{vmatrix} 2 & 3 & 4 \\ 1 & 0 & 2 \\ 5 & 0 & 3 \end{vmatrix}$

Expand along C_2

C_2 के अनुदिश विस्तार

$= -3\begin{vmatrix} 1 & 2 \\ 5 & 3 \end{vmatrix} + 0\begin{vmatrix} 2 & 4 \\ 5 & 3 \end{vmatrix} - 0\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix}$

$= -3 (3 - 10) + 0 - 0$

$= -3 \times (-7) = +21$ Ans

(c) $\Delta = \begin{vmatrix} 1 & 4 & 0 \\ 2 & 5 & 0 \\ 3 & 6 & 3 \end{vmatrix}$

Expand along C_3

C_3 के अनुदिश विस्तार

$$\Delta = +3\begin{vmatrix} 1 & 4 \\ 2 & 5 \end{vmatrix} = 3 \times (5 - 8)$$

$= 3 \times (-3)$

$= -9$ Ans

(d) $\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{vmatrix}$

$R_2 \longrightarrow 2R_2$

$$= \frac{1}{2}\begin{vmatrix} 2 & 4 & 6 \\ 2 & 4 & 6 \\ 3 & 1 & 0 \end{vmatrix}$$

$= \frac{1}{2} \times \mathbf{0} = 0 \quad \{\because R_1 = R_2\}$

(e) $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{vmatrix}$

$\because R_1 = R_2 = R_3$ या $C_1 = C_2 = C_3$

= 0 Ans

2. $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$

$\Rightarrow 12x - (-14) = 32 - 42$

$\Rightarrow 12x + 14 = -10$

$\Rightarrow 12x = -10 - 14$

$\Rightarrow 12X = -24$

$\Rightarrow X = -2$ Ans

3. $\Delta = \begin{vmatrix} 3 & -2 \\ 4 & 4 \end{vmatrix}$

The Co-factors of the elements of Δ are-

Δ के अवयवों के सहखण्ड हैं –

$A_{11} = +(4) = 4 \qquad A_{21} = -(-2) = 2$

$A_{12} = -4 \qquad A_{22} = +(3) = 3$

4. $\Delta = \begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$

$R_1 \longrightarrow R_1 + R_2 + R_3$

$$\Delta = \begin{vmatrix} 1+w+w^2 & 1+w+w^2 & 1+w+w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0 & 0 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0 \text{ Ans}$$

5. $\begin{vmatrix} x-9 & 1 \\ 7 & 1 \end{vmatrix} = \mathbf{0}$

$\Rightarrow (x - 9) \times 1 - 7 \times 1 = 0$

$\Rightarrow x - 9 - 7 = 0$

$\Rightarrow x - 16 = 0$

$\Rightarrow x = 16$ Ans

6. Given equations are :-

दिये गये समीकरण हैं –

$2x - y = 2$

$3x + 4y = 3$

Matrix form (आव्यूह रूप) $Ax = B$

$\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ Ans

7. $\begin{vmatrix} 5 & 60 & 7 \\ 3 & 36 & 2 \\ 2 & 24 & 4 \end{vmatrix}$ or $\begin{vmatrix} 5 & 60 & 7 \\ 3 & 36 & 2 \\ 2 & 24 & 4 \end{vmatrix}$

$C_1 \longrightarrow 12C_1$

$= \begin{vmatrix} 60 & 60 & 7 \\ 36 & 36 & 2 \\ 24 & 24 & 4 \end{vmatrix}$

$\because C_1 = C_2$

= 0

or

$C_2 \longrightarrow C_2 - 12C_1$

$= \begin{vmatrix} 5 & 0 & 7 \\ 3 & 0 & 2 \\ 2 & 0 & 4 \end{vmatrix}$

= 0

Short Questions (3 Marks)
लघुउत्तरीय प्रश्न

Q.1. Prove that (सिद्ध करें) $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ **= (a - b)(b - c)(c - a)**

Q.2. Prove that (सिद्ध करें) $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = \mathbf{0}$

Q.3. Prove that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$ **= (a -b)(b -c)(c -a)**

Q.4. Solve for x when, (x के लिए हल करें जब)

$$\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0$$

Q.5. Find the area of triangle whose vertices are

(1, 2), (2, 3), (3, 2)

शीर्षों (1, 2), (2, 3), (3, 2) वाले त्रिभुज का क्षेत्रफल ज्ञात करें।

Q.6. If $A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ -4 & 5 & 3 \end{bmatrix}$**, find adj A**

Q.7. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$**, verify that A. (adj A) = (adj A) A**

$= |A| . I$

Q.8. Find the inverse of the Matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

आव्यूह A का व्युत्क्रम ज्ञात करें।

Q.9. Prove that the Matrix $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ **satisfies the equation** $x^2 + 4x - 42 = 0$ **and hence find** A^{-1}.

सिद्ध करें कि आव्यूह $A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$ **समीकरण** $x^2 + 4x - 42 = 0$ **संतुष्ट करता है और** A^{-1}**ज्ञात करें।**

Q.10. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$**, Show that adj A = 3A'**

यदि $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$**, तो सिद्ध करें adj A = 3A'**

Solutions of short questions (3 Marks)

1. $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$, $R_1 \longrightarrow R_1 - R_2, R_2 \longrightarrow R_2 - R_3$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_1 \longrightarrow \frac{R_1}{(a-b)}, R_2 \longrightarrow \frac{R_2}{(b-c)}$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expand along C_1

(C_1 के अनुदिश विस्तार)

$$= (a-b)(b-c)\left\{+1\begin{vmatrix} 1 & a+b \\ 1 & b+c \end{vmatrix}\right\}$$

$= (a-b)(b-c)\{\not{b} + c - a - \not{b}\}$

$= (a-b)(b-c)(c-a)$ Proved

2. $L.H.S. = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

$R_1 \longrightarrow R_1 + R_2 + R_3$

$$= \begin{vmatrix} 0 & 0 & 0 \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$$

$= 0$

= R.H.S. Proved

3. $L.H.S. = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix}$

$C_1 \longrightarrow C_1 - C_2, C_2 \longrightarrow C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ bc-ca & ca-ab & ab \end{vmatrix}$$

$C_1 \longrightarrow \frac{C_1}{(a-b)}, C_2 \longrightarrow \frac{C_2}{(b-c)}$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -c & -a & ab \end{vmatrix}$$

$$= (a-b)(b-c)\left\{+1\begin{vmatrix} 1 & 1 \\ -c & -a \end{vmatrix}\right\}$$

$= (a-b)(b-c)(c-a)$

=R.H.S. Proved

4. $\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0$

$R_1 \longrightarrow R_1 - R_3$

$$\Rightarrow \begin{vmatrix} 1+x & 0 & -1-x \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix}$$

$C_3 \longrightarrow C_3 + C_1$

$$\Rightarrow \begin{vmatrix} 1+x & 0 & 0 \\ 1 & 7+x & 7 \\ 2 & 5 & 5+x \end{vmatrix} = 0$$

$$\Rightarrow (1+x)\begin{vmatrix} 7+x & 7 \\ 5 & 5+x \end{vmatrix} = 0$$

$$\Rightarrow (1+x)\{(7+x))(5+x) - 35\} = 0$$

$$\Rightarrow (1+x)(35 + 12x + x^2 - 35) = 0$$

$$\Rightarrow (1+x)(x^2 + 12x) = 0$$

$$\Rightarrow x(x+1)(x+12) = 0$$

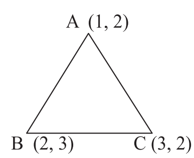
$x = 0, -1, -12$ Ans

5. Given Vertices are (दिए गए शीर्ष हैं)

$A = (1, 2) \quad x_1 = 1 \quad y_1 = 2$

$B = (2, 3) \quad x_2 = 2 \quad y_2 = 3$

$C = (3, 2) \quad x_3 = 3 \quad y_3 = 2$



$$ar(\triangle ABC) = \frac{1}{2}\left\|\begin{matrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{matrix}\right\|$$

$$= \frac{1}{2}\left\|\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 2 \end{matrix}\right\|$$

$$= \frac{1}{2}\left|1(4-9) - 1(2-6) + 1(3-4)\right|$$

$$= \frac{1}{2}\left|-5 + 4 - 1\right|$$

$$= \frac{1}{2}\left|-2\right|$$

$$= \frac{1}{2} x\ 2 = 1 \text{ units}^2 \text{ Ans}$$

6. $A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ -4 & 5 & 3 \end{bmatrix}$

Determinant (सारणिक) Of A

$$|A| = \begin{vmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ -4 & 5 & 3 \end{vmatrix}$$

The Co-factors of the elements of $|A|$ are

$|A|$ के अवयवों का सहखण्ड हैं :-

$$A_{11} = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = 6 - 5 = 1, A_{12} = -\begin{vmatrix} 0 & 1 \\ -4 & 3 \end{vmatrix} = -(0+4) = -4$$

$$A_{13} = \begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix} = 0 + 8 = 8$$

$$A_{21} = -\begin{vmatrix} -2 & 4 \\ 5 & 3 \end{vmatrix} = -(-6-20) = -(-26) = 26$$

$$A_{22} = +\begin{vmatrix} 1 & 4 \\ -4 & 3 \end{vmatrix} = (3 + 16) = 19$$

$$A_{23} = -\begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix} = -(5 - 8) = 3$$

$$A_{31} = \begin{vmatrix} -2 & 4 \\ 2 & 1 \end{vmatrix} = (-2 - 8) = -10$$

$$A_{32} = -\begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} = -(1 - 0) = -1$$

$$A_{33} = \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} = (2 - 0) = 2$$

$$\therefore \text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} 1 & -4 & 8 \\ 26 & 19 & 3 \\ -10 & -1 & 2 \end{bmatrix}'$$

$$= \begin{bmatrix} 1 & 26 & -10 \\ -4 & 19 & -1 \\ 8 & 3 & 2 \end{bmatrix} \text{Ans}$$

7. $A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}, |A| = \begin{vmatrix} 2 & 3 \\ 4 & 8 \end{vmatrix} = 16 - 12 = 4$

$$\text{adj } A = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

Now (अब)

$$A.(\text{adj } A) = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix} . \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16-12 & -6+6 \\ 32-32 & -12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= 4.\ I$

$= |A|.\ I$

$$(\text{adj } A).A = \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix} . \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 16-12 & 24-24 \\ -8+8 & -12+16 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$= 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= 4.\ I$

$= |A|.\ I$

Clearly $A.(\text{adj } A) = (\text{adj } A) . A = |A| . I$ <u>Proved</u>

8. $$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = -(-1)\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = 9 + 2 = 11$$

The Co-factors of the elements of $|A|$ are

$A_{11} = 0$, $A_{12} = -11$, $A_{13} = 0$

$A_{21} = 3$, $A_{22} = 1$, $A_{23} = -1$

$A_{31} = 2$, , $A_{32} = 8$, $A_{33} = 3$

$$\text{adj } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

Now, Inverse of A

$$A^{-1} = \frac{1}{|A|} . \text{adj } A$$

$$= \frac{1}{11}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \text{Ans.}$$

9. $$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

Given equation (दिया गया समीकरण)

$x^2 + 4x - 4 = 0$

Put $x = A$

$A.A + 4A - 42.I = 0$

$$\begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} . \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + 4 . \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} - 42\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} + \begin{bmatrix} -42 & 0 \\ 0 & -42 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -74 & 20 \\ 8 & -26 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$\Rightarrow 0 = 0$ (True) (सत्य) Proved

Now From equation (1)

$A^2 + 4A - 42.I = 0$

$A^{-1} . (A^2 + 4A - 42.I) = A^{-1} . 0$

$\Rightarrow A + 4.I - 42A^{-1} = 0$

$\Rightarrow 42A^{-1} = A + 4.I$

$$\Rightarrow 42A^{-1} = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$\Rightarrow 42A^{-1} = \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix} \quad \therefore A^{-1} = \frac{1}{42}\begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix} \text{Ans}$$

10. $$A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

The Co-factors of the elements of $|A|$ are

$A_{11} = -3$, $A_{12} = -6$, $A_{13} = -6$

$A_{21} = 6$, $A_{22} = 3$, $A_{23} = -6$

$A_{31} = 6$, , $A_{32} = -6$, $A_{33} = 3$

$$\text{adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\text{and } A' = \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix}$$

$$3\,A' = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

Clearly $\text{adj}A = 3A'$ Proved

Long Questions (5 Marks)
दीर्घउत्तरीय प्रश्न

1. **If** $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$**, Find** A^{-1} **and hence solve the system of linear equations.**

$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11$

यदि $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$ तो A^{-1} ज्ञात करें और निम्नलिखित रैखिक समीकरण निकाय का हल करें।

$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2,\ 3x - 3y - 4z = 11$

2. **By using elemntry row transformation, find the inverse of the matrix** $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

प्रारंभिक पंक्ति संक्रिया से वर्ग आव्यूह $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ का व्युत्क्रम ज्ञात करें।

3. **Prove that** $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ **is neither a symmetric matrix nor a skew-symetric matrix.**

Also express A as the sum of a symmetric Matrix and a skew-symmetric Matrix.

सिद्ध करें कि $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ न तो सममित है और न ही विषम सममित है। फिर A को एक सममित आव्यूह और विषम सममित आव्यूह के योगफल के रूप में व्यक्त करें।

4. **The sum of three numbers is 6. Twice the third number when added to the first number gives 7 .On adding the sum of the second and third numbers to the thrice the first number we get 12. Find the numbers, using Matrix Method.**

तीन संख्याओं का योग 6 है। जब पहले संख्या में तीसरे संख्या का दो गुना जोड़ने पर 7 प्राप्त होता है। दूसरे तथा तीसरे संख्याओं में पहले संख्या का तीन गुणा जोड़ने पर 12 प्रप्त होता है। तीनों संख्याओं को आव्यूह विधि के द्वारा ज्ञात करें।

5. **Prove that (सिद्ध करें)**

$$\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$

Solutions of long questions

1. Given system of equations are -

$x + 2y - 3z = -4$ ----------(i)

$2x + 3y + 2z = 2$ ----------(ii)

$3x - 3y - 4z = 11$ ----------(iii)

the given system of equations in matrix form is AX = B

$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$,

$= 1(-12 + 6) - 2(-8 - 6) - 3(-6 - 9)$

$= -6 + 28 + 45$

$= -6 + 73$

$= 67 \neq 0$

$\therefore$ A is invertible (A व्युतक्रमणीय है)

So, the system has a unique solution

$x = A^{-1}B$................(1)

Now, the Co-factors of the elements of $|A|$ are

$|A|$ के अवयवों का सहखण्ड हैं :-

$A_{11} = -6$, $A_{12} = 14$, $A_{13} = -15$

$A_{21} = 17$, $A_{22} = 5$, $A_{23} = 9$

$A_{31} = 13$, $A_{32} = -8$, $A_{33} = -1$

$\therefore \text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$

and $A^{-1} = \frac{1}{|A|} \text{adj } A$

$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

Now from equation (1) (अब समीकरण (1) से)

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}.\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}.\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}.\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

So, $x = 3$, $y = -2$, $z = 1$ Ans

2. Given, $\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

we know A= I.A

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 5 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$R_2 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & -2 & -5 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & -2 & 0 \end{bmatrix}.A$$

$R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & 1 \\ 5 & -2 & 2 \end{bmatrix}.A$$

$R_1 \rightarrow R_1 + R_3$, $R_2 \rightarrow R_2 - 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}.A$$

Hence, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ Ans

3. Given, $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Now (अब) $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Here (यहाँ) $A \neq A' \Rightarrow$ Not symmetric Matrix
सममित आव्यूह नहीं है।

$A \neq -A' \Rightarrow$ Not Skew-symmetric Matrix
विषम सममित आव्यूह नहीं है।

Symmetric Matrix $= \frac{1}{2}(A + A')$
(सममित आव्यूह)

$$= \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

Skew-symmetric Matrix $= \frac{1}{2}(A - A')$
(विषम सममित आव्यूह)

$$= \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

square Matrix, A = symmetric Matrix + Skew symmetric Matrix

$$A = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

Ans

4. Let First number = x
पहली संख्या = x

Second Number = y

दूसरी संख्या = y

Third Number = z

तीसरी संख्या = z

By question (प्रश्न से)

$x + y + z = 6$ ---------(1)

$x + 2z = 7$ ----------(2)

$3x + y + z = 12$ -------(3)

The Given system in Matrix form is AX =B

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$|A| = 1(0 - 2) - 1(1 - 6) + 1(1 - 0)$

$= -2 + 5 + 1$

$|A| = 4 \neq 0$ $\therefore$ A is invertible (A व्युत्क्रमणीय है।)

Now $X = A^{-1}B$ ---------(4)

The Co-factors of the elements of $|A|$ are-

$A_{11} = -2$, $A_{12} = 5$, $A_{13} = 1$

$A_{21} = 0$, $A_{22} = -2$, $A_{23} = 2$

$A_{31} = 2$, $A_{32} = -1$, $A_{33} = -1$

$$\therefore \text{adj } A = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$\text{Now } A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$= \frac{1}{4} \cdot \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$\therefore$ By (4), $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \cdot \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4} \cdot \begin{bmatrix} -12+0+24 \\ 30-14-12 \\ 6+14-12 \end{bmatrix}$$

$$= \frac{1}{4} \cdot \begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ $x = 3, y = 1, z = 2$

$\therefore$ Required numbers = 3, 1, 2 Ans

(अभिष्ट संख्याएँ)

5. $$\text{L.H.S.} = \begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$$

$C_1 \to \frac{C_1}{a}, C_2 \to \frac{C_2}{b}, C_3 \to \frac{C_3}{c},$

$$= abc \begin{vmatrix} a & c & c+a \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

$R_2 \to R_2 - (R_1 + R_3)$

$$= abc \begin{vmatrix} a & c & c+a \\ 0 & -2c & -2c \\ b & b+c & c \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$= abc \begin{vmatrix} a & -a & c+a \\ 0 & 0 & -2c \\ b & b & c \end{vmatrix}$$

Expand along R_2

(R_2 के अनुदिश विस्तार करने पर)

$$= abc \,.\, 2c \begin{vmatrix} a & -a \\ b & b \end{vmatrix}$$

$= 2abc^2 . (ab + ab)$

$= 2abc^2 \times 2ab$

$= 4 a^2b^2c^2$

= R.H.S. Proved

Chapter: 5 — Continuity and Differentiability
सांतत्य तथा अवकलनीयता

बहुविकल्पीय प्रश्न (MCQ)

1 Marks Question:-

1. **If $f(x) = x^2 + 2x$ then f(2)=?**
यदि $f(x) = x^2 + 2x$ तो f(2)=?
(a) 4 (b) 8
(c) 2 (d) 6

2. **A function $f(x) = x^2$ is continuous at**
(a) 2 (b) 3
(c) 0 (d) All of the above
फलन $f(x) = x^2$ संतत होता है
(a) 2 पर (b) 3 पर
(c) 0 पर (d) उपरोक्त सभी

3. **The function** $f(x) = \begin{cases} 2x - 1, & x<0 \\ 2x + 1, & x\geq 0 \end{cases}$ **is discontinuous at**
(a) 1 (b) 2
(c) 0 (d) None of these
फलन $f(x) = \begin{cases} 2x - 1, & x<0 \\ 2x + 1, & x\geq 0 \end{cases}$ **संतत नहीं है**
(a) 1 पर (b) 2 पर
(c) 0 पर (d) इनमें से कोई नहीं

4. **Function f(x) is continuous at x=a if**
फलन f(x), x=a पर संतत है यदि
(a) $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$
(b) $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x) = f(a)$
(c) $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) \neq f(a)$
(d) None of these इनमें से कोई नहीं

5. **प्रत्येक नियत फलन होता है।**
Every Constant function is
(a) Discontinuous (असंतत)
(b) Continuous (संतत)
(c) Discontinuous and Continuous (असंतत और संतत)
(d) None of these (इनमें से कोई नहीं)

6. **Which of the following function is continuous**
निम्नलिखित फलन मे से कौन सा संतत है–
(a) Identity function (तत्समक फलन)
(b) Polynomial function (बहुपद फलन)
(c) Rational function (परिमेय फलन)
(d) All of the above (उपरोक्त सभी)

7. **If f and g be continuous function then**
यदि f और g संतत फलन है, तो
(a) f+g is also continuous (f+g भी संतत हैं)
(b) f.g is also continuous (f.g भी संतत हैं)
(c) Both (a) and (b)/ दोनो (a) तथा (b)
(d) None of these./ इनमें से कोई नहीं।

8. **The value of k for which**
$f(x) = \begin{cases} \frac{\sin 5x}{3x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$
is continuous at x = 0
यदि x = 0 पर $f(x) = \begin{cases} \frac{\sin 5x}{3x}, & \text{यदि } x \neq 0 \\ k, & \text{यदि } x = 0 \end{cases}$
संतत है तो k का मान होगा–
(a) $\frac{1}{3}$ **(b) 0**
(c) $\frac{3}{5}$ **(d)** $\frac{5}{3}$

9. **Let** $f(x) = x^{3/2}$. **Then** f'(o) = ?
माना $f(x) = x^{3/2}$. f'(o) = ?
(a) $\frac{3}{2}$ (b) $\frac{1}{2}$
(c) does not exist (प्राप्त नहीं है) (d) None of these (इनमें से कोई नहीं)

10. **The function f(x) = |x|, $\forall x \in R$ is**
फलन f(x) = |x|, $\forall x \in R$ है–
(a) Continuous but not differentiable at x=0
x=o पर संतत परन्तु अवकलनीय नहीं
(b) differentiable but not continuous at x=0
x=0 पर अवकलनीय परन्तु संतत नहीं
(c) Neither continuous nor differentiable at x=0
x=0 पर, ना संतत ना अवकलनीय
(d) None of these . (इनमें से कोई नहीं)

11. **If the function** $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq\pi/2\\ 3, & x=\pi/2\end{cases}$ **be continuous at** $x=\pi/2$ **then the value of k is**

यदि $x=\pi/2$ पर $f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & x\neq\pi/2\\ 3, & x=\pi/2\end{cases}$ संतत है तो **k** का मान है।

(a) -3 (b) -5
(c) 6 (d) 3

12. **If** $y=\sin(x^2+5)$ **then** $\frac{dy}{dx}=?$

यदि $y=\sin(x^2+5)$ तो $\frac{dy}{dx}=?$

(a) $2x.\sin(x^2+5)$ (b) $2x.\cos(x^2+5)$
(c) $x.\cos(x^2+5)$ (d) None

13. **If** $y=\cos(\sin x)$ **then** $\frac{dy}{dx}=?$

(a) $-\cos x.\sin(\sin x)$ (b) $\cos x.\sin(\sin x)$
(c) $-\sin(\sin x)$ (d) $-\cos(\sin x)$

14. **If** $y=x^2$ **then** $\frac{dy}{dx}=?$

(a) x (b) 2x
(c) -2 (d) -x

15. **if** $y=x^{-3}$ **then** $\frac{dy}{dx}=?$

(a) $-3x^{-4}$ (b) $3x^4$
(c) $3x^2$ (d) None

16. **If** $y=\cos(\sqrt{x})$ **then** $\frac{dy}{dx}=?$

(a) $\frac{\text{Sin}\sqrt{x}}{2\sqrt{x}}$ (b) $\frac{-\text{Sin}\sqrt{x}}{\sqrt{x}}$
(c) $\frac{-\text{Sin}\sqrt{x}}{2\sqrt{x}}$ (d) $\frac{\text{Cos}\sqrt{x}}{2\sqrt{x}}$

17. **If** $2x+3y=\sin x$ **then** $\frac{dy}{dx}=?$

(a) cos x (b) -cos x
(c) $\frac{\cos x+2}{3}$ (d) $\frac{\cos x-2}{3}$

18. **If** $y=e^{x^3}$ **then** $\frac{dy}{dx}=?$

(a) $3x^2.e^{x^3}$ (b) $x^2.e^{x^3}$
(c) $3x^2.e^{x^2}$ (d) None

19. **If** $y=\sin x^3$ **then** $\frac{dy}{dx}=?$

(a) $\sin x^3$ (b) $-\cos x^3$
(c) $3x^2.\cos x^3$ (d) 0

20. **If** $y=\log\sin x$ **then** $\frac{dy}{dx}=?$

(a) cot x (b) sin x
(c) tan x (d) -cot x

21. **If** $y=3^x$ **then** $\frac{dy}{dx}=?$

(a) 3^x (b) log 3
(c) $3^x.\log 3$ (d) None

22. **If** $y=\sin^{-1}(x^2)$ **then** $\frac{dy}{dx}=?$

(a) $\frac{2x}{\sqrt{x^4-1}}$ (b) $\frac{2x}{\sqrt{x^4+1}}$
(c) $\frac{x}{\sqrt{x^2-1}}$ (d) $\frac{1}{\sqrt{x^4-1}}$

23. **If** $y=\tan^{-1}(x^3)$ **then** $\frac{dy}{dx}=?$

(a) $\frac{3x^2}{x^2-1}$ (b) $\frac{x^2}{x^2+1}$
(c) $\frac{x^2}{x-1}$ (d) $\frac{3x^2}{x^6+1}$

24. **If** $y=\cos^{-1}x^3$ **then** $\frac{dy}{dx}=?$

(a) $\frac{-1}{1+x}$ (b) $\frac{2}{\sqrt{1+x}}$
(c) $\frac{-1}{2\sqrt{x}(1+x)}$ (d) None

25. **If** $y=\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ **then** $\frac{dy}{dx}=?$

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$
(c) 1 (d) -1

26. **If** $x=at^2$, $y=2at$ **then** $\frac{dy}{dx}=?$

(a) $\frac{1}{t}$ (b) $\frac{-1}{t^2}$
(c) $\frac{-2}{t}$ (d) None

27. **If** $x=a\sec\theta$, $y=b\tan\theta$ **then** $\frac{dy}{dx}=?$

(a) $\frac{b}{a}\sec\theta$ (b) $\frac{b}{a}\text{cosec}\,\theta$
(c) $\frac{b}{a}\cot\theta$ (d) None

28. **if** $x=t$, $y=2t$ **then** $\frac{dy}{dx}=?$

(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) None

29. **If** $y=\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ **then** $\frac{dy}{dx}=?$

(a) 1 (b) -1
(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

30. **If** $y=\sqrt{x^5}$ **then find** $\frac{dy}{dx}=?$

(a) $5x^4$ (b) $\frac{5x^4}{\sqrt{x^5}}$
(c) $\frac{5x^4}{2\sqrt{x^5}}$ (d) None

MCQ Answers
बहुविकल्पीय प्रश्न उत्तर

1. b	11. c	21. c
2. d	12. b	22. a
3. c	13. a	23. d
4. a	14. b	24. d
5. b	15. a	25. c
6. d	16. c	26. a
7. c	17. d	27. b
8. d	18. a	28. b
9. c	19. c	29. c
10. a	20. a	30. c

अति लघु उत्तरीय प्रश्न (Very Short Questions) (2 Marks)

1. **Prove that the function f(x) = 5x-3 is continuous at x=0.**

   सिद्ध करें कि x=0 पर फलन f(x) = 5x-3 संतत है।

2. **Show that the function $f(x)=x^2$ is differentiable at x = 1 and find f'(1)**

   सिद्ध करें x = 1 पर फलन $f(x) = x^2$ अवकलनीय है और f'(1) ज्ञात करें।

3. **If** $y = \sin 4x$ **then find** $\frac{dy}{dx}$.

   यदि $y = \sin 4x$ तो $\frac{dy}{dx}$ ज्ञात करें।

4. **If** $y = \cot\sqrt{x}$ **then find** $\frac{dy}{dx}$.

5. **If** $y = \sqrt{\sin x}$ **then find** $\frac{dy}{dx}$.

6. **If** $y = \sin 5x.\cos 3x$ **then find** $\frac{dy}{dx}$

7. **If** $y = \sqrt{x}$ **then find** $\frac{dy}{dx}$.

8. **If** $y = (2x + 3)^5$ **then find** $\frac{dy}{dx}$.

9. **If** $y = e^{x^2}$ **then find** $\frac{dy}{dx}$.

10. **If** $y = \log(2x + 1)$ **then find** $\frac{dy}{dx}$.

11. **If** $y = e^{3x}$ **then find** $\frac{dy}{dx}$.

12. **If** $y = \cos\frac{x}{2}$ **then find** $\frac{dy}{dx}$.

13. **If** $y = x.\sin x$ **then find** $\frac{dy}{dx}$.

14. **If** $y = \tan^{-1}x^2$ **then find** $\frac{dy}{dx}$.

15. **If** $y = \cos^{-1}2x$ **then find** $\frac{dy}{dx}$.

16. **If** $y = \log(2x+3)$ **then find** $\frac{dy}{dx}$.

17. **If** $y = \text{cosec}(\sqrt{x})$ **then find** $\frac{dy}{dx}$.

18. **If** $y = e^{\sin x}$ **then find** $\frac{dy}{dx}$.

19. **If** $y = \sec^{-1}(x^2)$ **then find** $\frac{dy}{dx}$.

20. **If** $y = (3x^2 + 2x + 1)^2$ **then find** $\frac{dy}{dx}$.

Solutions of very short questions
अति लघु उत्तरीय प्रश्न का हल

1. Given function $f(x) = 5x - 3$

at $x = 0$

$f(0) = 5(0)-3 = -3$

$$\lim_{x\to0^+} f(x) = \lim_{h\to0} f(0+h) = \lim_{h\to0} 5(0+h) - 3 = \lim_{h\to0} 5h - 3 = 5(0) - 3 = -3$$

$$\lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h) = \lim_{h\to0} 5(0-h) - 3 = \lim_{h\to0} -5h - 3 = -5(0) - 3 = -3$$

Clearly $\lim_{x\to0^+} f(x) = \lim_{x\to0^-} f(x) = f(0)$

$\therefore$ f (x) is continuous at x = 0

x = 0 पर f (x) संतत है

2. Given function $f(x) = x^2$

at $x = 1$

$$R f'(1) = \lim_{h\to0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to0} \frac{(1+h)^2 - (1)^2}{h} = \lim_{h\to0} \frac{1 + 2h + h^2 - 1}{h} = \lim_{h\to0} \frac{2h + h^2}{h} = \lim_{h\to0} (2 + h) = 2 + 0 = 2$$

$$\text{and } L f'(1) = \lim_{h\to0} \frac{f(1-h) - f(1)}{-h} = \lim_{h\to0} \frac{(1-h)^2 - (1)^2}{-h} = \lim_{h\to0} \frac{1-2h+h^2-1}{-h} = \lim_{h\to0} \frac{h^2-2h}{-h} = \lim_{h\to0} \frac{h(h-2)}{-h} = 2$$

$\therefore$ R f' (1) = Lf'(1) = 2

$\therefore$ f(x) is differentiable at x = 1 and f'(1) = 2

3. Given $y = \sin 4x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sin 4x}{dx}$$
$$= \cos 4x \times 4 \times (1)$$
$$= 4 \cos 4x$$

4. Given $y = \cot\sqrt{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cot\sqrt{x}}{dx}$$
$$= \frac{d \cot\sqrt{x}}{d\sqrt{x}} \times \frac{d\sqrt{x}}{dx}$$
$$= -\operatorname{cosec}^2\sqrt{x} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{-\operatorname{cosec}^2\sqrt{x}}{2\sqrt{x}}$$

5. Given $y = \sqrt{\sin x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{\sin x}}{dx}$$
$$= \frac{d\sqrt{\sin x}}{d \sin x} \times \frac{d \sin x}{dx}$$
$$= \frac{1}{2\sqrt{\sin x}} \times \cos x$$
$$= \frac{\cos x}{2\sqrt{\sin x}}$$

6. Given $y = \sin 5x \cdot \cos 3x$

$$y = \frac{1}{2} \cdot 2 \sin 5x \cdot \cos 3x$$
$$y = \frac{1}{2}\left[\sin(5x + 3x) + \sin(5x-3x)\right]$$
$$y = \frac{1}{2}\left[\sin 8x + \sin 2x\right]$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{d \sin 8x}{dx} + \frac{d \sin 2x}{dx}\right]$$
$$\frac{dy}{dx} = \frac{1}{2}\left[\cos 8x \times 8 + \cos 2x \times 2\right]$$
$$\frac{dy}{dx} = 4 \cos 8x + \cos 2x$$

7. Given $y = \sqrt{x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{x}}{dx}$$
$$= \frac{1}{2\sqrt{x}}$$

8. Given $y = (2x + 3)^5$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\,(2x + 3)^5}{dx}$$
$$= \frac{d\,(2x + 3)^5}{d\,(2x + 3)} \times \frac{d\,(2x + 3)}{dx}$$
$$= 5(2x + 3)^4 \times [2 \cdot (1) + 0]$$
$$= 10(2x + 3)^4$$

9. Given $y = e^{x^2}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, e^{x^2}}{dx}$$
$$= \frac{d\, e^{x^2}}{dx^2} \times \frac{dx^2}{dx}$$
$$= e^{x^2} \times 2x$$
$$= 2x \cdot e^{x^2}$$

10. Given $y = \log(2x + 1)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \log(2x+1)}{dx}$$
$$= \frac{d \log(2x+1)}{d(2x+1)} \times \frac{d(2x+1)}{dx}$$
$$= \frac{1}{(2x+1)} \times (2 + 0)$$
$$= \frac{2}{2x+1}$$

11. Given $y = e^{-3x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, e^{-3x}}{dx}$$
$$= \frac{d\, e^{-3x}}{d(-3x)} \times \frac{d(-3x)}{dx}$$
$$= e^{-3x} \times (-3)$$
$$= -3 \cdot e^{-3x}$$

12. Given $y = \cos\frac{x}{2}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cos\frac{x}{2}}{dx}$$
$$= \frac{d \cos x/2}{d x/2} \times \frac{d\, x/2}{dx}$$
$$= -\sin\frac{x}{2} \times \frac{1}{2}$$
$$= -\frac{1}{2}\sin\frac{x}{2}$$

13. Given $y = x.\sin x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, x.\sin x}{dx}$$
$$= x \cdot \frac{d \sin x}{dx} + \sin x. \frac{d\,x}{dx}$$
$$= x \cdot (\cos x) + \sin x.(1)$$
$$= x \cdot \cos x + \sin x$$

14. Given $y = \tan^{-1} x^2$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \tan^{-1} x^2}{dx}$$
$$= \frac{d \tan^{-1} x^2}{dx^2} \times \frac{dx^2}{dx}$$
$$= \frac{1}{1+(x^2)^2} \times 2x$$
$$= \frac{2x}{1+x^4}$$

15. Given $y = \cos^{-1} 2x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cos^{-1} 2x}{dx}$$
$$= \frac{d \cos^{-1} 2x}{d2x} \times \frac{d\,2x}{dx}$$
$$= \frac{d \cos^{-1} 2x}{d2x} \times \frac{d\,2x}{dx}$$
$$= \frac{-1}{\sqrt{1-(2x)^2}} \times 2$$
$$= \frac{-2}{\sqrt{1-4x^2}}$$

16. Given $y = \log(2x+3)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \log(2x+3)}{dx}$$
$$= \frac{d \log(2x+3)}{d(2x+3)} \times \frac{d(2x+3)}{dx}$$
$$\frac{dy}{dx} = \frac{1}{2x+3} \times 2$$
$$= \frac{2}{2x+3}$$

17. Given $y = \operatorname{cosec}(\sqrt{x})$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \operatorname{cosec}(\sqrt{x})}{dx}$$
$$= \frac{-\operatorname{cosec}\sqrt{x} \cdot \cot\sqrt{x}}{2\sqrt{x}}$$

18. Given $y = e^{\sin x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\, e^{\sin x}}{dx}$$
$$= e^{\sin x} \cdot \cos x$$

19. Given $y = \sec^{-1}(x^2)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sec^{-1}(x^2)}{dx}$$
$$= \frac{1}{x^2 \cdot \sqrt{x^4-1}} \times 2x$$
$$= \frac{2}{x\sqrt{x^4-1}}$$

20. Given $y = (3x^2 + 2x + 1)^2$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d(3x^2 + 2x + 1)^2}{dx}$$
$$= 2(3x^2 + 2x + 1) \cdot (6x + 2)$$
$$= 4(3x + 1) \cdot (3x^2 + 2x + 1)$$

Short Question (लघु उत्तरीय प्रश्न)
(3 Marks)

1. **Discuss the continuity of the function f(x) at x = 0 if**

$$f(x) = \begin{cases} 2x - 1, & x<0 \\ 2x + 1, & x \geq 0 \end{cases}$$

2. **Find the value of k for which**

$$f(x) = \begin{cases} kx + 5, & x \leq 2 \\ x-1, & x > 2 \end{cases}$$

is continuous at x = 2

3. **Find the value of k for which**

$$f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 2 \\ k, & x = 0 \end{cases}$$

is continuous at x = 2,

$$\text{यदि}, f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 2 \\ k, & x = 0 \end{cases}$$

x = 2 पर संतत है तो k का मान ज्ञात करें।

4. **Show that the function** $f(x) = \begin{cases} 1+x, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$ **is not differentiable at** $x = 2$

सिद्ध करें कि फलन $f(x) = \begin{cases} 1+x, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$ $x = 2$ पर अवकलनीय नहीं है।

Find $\frac{dy}{dx}$ ($\frac{dy}{dx}$ **ज्ञात करें**)

5. $y = \cot^3 x^2$

6. $y = \sqrt{\sin x^3}$

7. $y = \sin(\log x)$

8. $y = \sqrt{e^{\sqrt{x}}}$

9. $y = \cos^{-1}(\cot x)$

10. $y = \sin(\tan^{-1} x)$

11. $y = \log(\tan^{-1}x)$

12. $x^2 + y^2 = 4$

13. $x = at^2$, $y = 2at$

14. $y = x^{10} + 2x^5 + \sqrt{\sin x}$

15. $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

16. $y = \cos^{-1}\sqrt{1-x^2}$

17. $y = x^x$

18. $y = b\sin\theta$, $x = a\cos\theta$

19. $y = \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

20. $y = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

Solutions of short questions (लघु उत्तरीय प्रश्न का हल)

1. Given $f(x) = \begin{cases} 2x - 1, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$

at x = 0 (x = 0 **पर**)

$f(0) = 2\cdot(0)+1 = 1$

$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h)$

$= \lim_{h\to 0} 2(0+h) + 1$

$= 1$

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h)$

$= \lim_{h\to 0} 2(0-h) + (-1)$

$= -1$

Clearly $f(0) = \lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

$\therefore$ f(x) is discontinuous at x = 0

x = 0 **पर** f(x) **असंतत है।**

2. **Given,** $f(x) = \begin{cases} kx + 5, & x \leq 2 \\ x - 1, & x > 2 \end{cases}$

f(x) is continuous at x = 2

x = 2 पर f(x) संतत है

$\therefore f(2) = \lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

$\Rightarrow k\cdot(2) + 5 = \lim_{h\to 0} f(2+h) = \lim_{h\to 0} f(2-h)$

$\Rightarrow 2k + 5 = \lim_{h\to 0} (2+h) - 1 = \lim_{h\to 0} k(2-h) + 5$

$\Rightarrow 2k + 5 = \lim_{h\to 0} (h + 1)$

$\Rightarrow 2k + 5 = 1$

$\Rightarrow 2k = 1 - 5$

$\Rightarrow 2k = -4$

$\Rightarrow k = -2$

3. **Given,** $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$

f(x) is continuous at x = 0

x = 0 पर f(x) संतत है।

$\therefore f(0) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$

$\Rightarrow k = \lim_{h\to 0} f(0+h)$

$\Rightarrow k = \lim_{h\to 0} \frac{1 - \cos(4(0+h))}{8(0+h)^2}$

$\Rightarrow k = \lim_{h\to 0} \frac{1 - \cos 4h}{8h^2}$

$\Rightarrow k = \lim_{h\to 0} \frac{2\sin^2 2h}{8h^2}$

$\Rightarrow k = \lim_{h\to 0} \frac{\sin^2 2h}{4h^2}$

$\Rightarrow k = \lim_{h\to 0} \left(\frac{\sin 2h}{2h}\right)^2$

$\Rightarrow k = (1)^2$

$\Rightarrow k = 1$

4. **Given,**

$f(x) = \begin{cases} 1 + x, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$

at x = 2 (x=2 **पर**)

$Rf'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h\to 0} \frac{[5-(2+h)] - [1+2]}{h}$

$= \lim_{h\to 0} \frac{5-2-h-3}{h}$

$= \lim_{h\to 0} \frac{-h}{h}$

$= -1$

$Lf'(2) = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h}$

$$= \lim_{h\to 0} \frac{[1+(2-h)] - [1+2]}{-h}$$
$$= \lim_{h\to 0} \frac{1+2-h-3}{-h}$$
$$= \lim_{h\to 0} \frac{-h}{-h}$$
$$= 1$$

Clearly $Rf'(2) \neq Lf'(2)$

$\therefore f(x)$ is not differentiable at $x = 2$

$x = 2$ पर $f(x)$ अवकलनीय नहीं है।

5. **Given,**

$y = \cot^3 x^2 = (\cot x^2)^3$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d(\cot x^2)^3}{dx}$$
$$= \frac{d(\cot x^2)^3}{d(\cot x^2)} \times \frac{d \cot x^2}{dx^2} \times \frac{dx^2}{dx}$$
$$= 3(\cot x^2)^2 \times (-\text{cosec}^2 x^2) \times (2x)$$
$$= -6x \cdot \cot^2 x^2 \cdot \text{cosec}^2 x^2$$

6. Given

$y = \sqrt{\sin x^3}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{\sin x^3}}{dx}$$
$$= \frac{d\sqrt{\sin x^3}}{d \sin x^3} \times \frac{d \sin x^3}{dx^3} \times \frac{dx^3}{dx}$$
$$= \frac{1}{2\sqrt{\sin x^3}} \times \cos x^3 \times 3x^2$$
$$= \frac{3x^2 \cdot \cos x^3}{2\sqrt{\sin x^3}}$$

7. Given

$y = \sin(\log x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sin(\log x)}{dx}$$
$$= \frac{d \sin(\log x)}{d(\log x)} \times \frac{d \log x}{dx}$$
$$= \cos(\log x) \times \frac{1}{x}$$
$$= \frac{\cos(\log x)}{x}$$

8. Given

$y = \sqrt{e^{\sqrt{x}}}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\sqrt{e^{\sqrt{x}}}}{dx}$$
$$= \frac{d\sqrt{e^{\sqrt{x}}}}{de^{\sqrt{x}}} \times \frac{d e^{\sqrt{x}}}{d\sqrt{x}} \times \frac{d\sqrt{x}}{dx}$$
$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} \times e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}}$$
$$= \frac{e^{\sqrt{x}}}{4\sqrt{x} \cdot \sqrt{e^{\sqrt{x}}}}$$
$$= \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$$

9. Given

$y = \cos^{-1}(\cot x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \cos^{-1}(\cot x)}{dx}$$
$$= \frac{d \cos^{-1}(\cot x)}{d \cot x} \times \frac{d \cot x}{dx}$$
$$= \frac{-1}{\sqrt{1-\cot^2 x}} \times (-\text{cosec}^2 x)$$
$$= \frac{\text{cosec}^2 x}{\sqrt{1-\cot^2 x}}$$

10. Given

$y = \sin(\tan^{-1} x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \sin(\tan^{-1} x)}{dx}$$
$$= \frac{d \sin(\tan^{-1} x)}{d \tan^{-1} x} \times \frac{d \tan^{-1} x}{dx}$$
$$= \cos(\tan^{-1} x) \times \frac{1}{1+x^2}$$
$$= \frac{\cos(\tan^{-1} x)}{1+x^2}$$

11. Given $y = \log(\tan^{-1} x)$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d \log(\tan^{-1} x)}{dx}$$
$$= \frac{d \log(\tan^{-1} x)}{d \tan^{-1} x} \times \frac{d \tan^{-1} x}{dx}$$
$$= \frac{1}{\tan^{-1} x} \times \frac{1}{1+x^2} = \frac{1}{(1+x^2)\tan^{-1} x}$$

12. Given $x^2+y^2 = 4$

d.w.r. to x

$$2x + 2y\frac{dy}{dx} = 0$$
$$\Rightarrow \quad 2y\frac{dy}{dx} = -2x$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{-x}{y}$$

13. Given

$x = at^2$ $\qquad$ $x = 2at$

d.w.r. to t $\qquad$ d.w.r. to t

$$\frac{dx}{dt} = 2at\ , \quad \frac{dy}{dt} = 2a$$

$$\text{Now}\quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{2a}{2at} = \frac{1}{t}$$

14. Given $y = x^{10} + 2x^5 + \sqrt{\sin x}$

d.w.r. to x

$$\frac{dy}{dx} = \frac{d\,x^{10}}{dx} + \frac{d\,2x^5}{dx} + \frac{d\sqrt{\sin x}}{dx}$$

$$= 10\,x^9 + 10\,x^4 + \frac{d\sqrt{\sin x}}{d\sin x} \times \frac{d\sin x}{dx}$$

$$= 10\,x^9 + 10\,x^4 + \frac{1}{2\sqrt{\sin x}} \times \cos x$$

$$= 10\,x^9 + 10\,x^4 + \frac{\cos x}{2\sqrt{\sin x}}$$

15. Given

$$y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Put $x = \tan\theta$ $\quad \therefore\ \theta = \tan^{-1} x$

$$y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$y = \tan^{-1}(\tan 2\theta)$

$y = 2\theta = 2\tan^{-1}x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

16. Given

$y = \cos^{-1}\sqrt{1-x^2}$

Put $x = \sin\theta$ $\quad \therefore\ \theta = \sin^{-1} x$

$y = \cos^{-1}\sqrt{1-\sin^2\theta}$

$y = \cos^{-1}\sqrt{\cos^2\theta}$

$y = \cos^{-1}(\cos\theta)$

$y = \theta$

$y = \text{Sin}^{-1}x$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

17.

Given

$y = x^x$

taking log both sides

दोनों पक्षों में log लेने पर

$\log y = \log x^x$

$\log y = x.\log x$

d.w.r. to x

$$\frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{1}{x} + \log x\cdot 1$$

$$\frac{dy}{dx} = y[1+\log x]$$

$$= x^x(1+\log x)$$

18. Given

$x = a\cos\theta$ $\qquad$ $y = b\sin\theta$

d.w.r. to θ $\qquad$ d.w.r. to θ

$$\frac{dx}{d\theta} = -a\sin\theta,\quad \frac{dy}{d\theta} = b\cos\theta$$

$$\therefore\ \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$= \frac{b\cos\theta}{-a\sin\theta}$$

$$= \frac{-b}{a}\ .\cot\theta$$

19. Given

$$y = \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\right)$$

$$\Rightarrow y = \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \frac{x}{2}$$

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{2}$$

20. Given

$y = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

put $x = \sin\theta \quad \therefore \quad \theta = \sin^{-1} x$

$y = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2\theta}}\right)$

$y = \sec^{-1}\left(\frac{1}{\sqrt{\cos^2\theta}}\right)$

$y = \sec^{-1}\left(\frac{1}{\cos\theta}\right)$

$y = \sec^{-1}(\sec\theta)$

$y = \theta$

$y = \sin^{-1} x$

d.w.r. to x

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

लघु उत्तरीय प्रश्न (Short Question)

3 Marks Question:-

Find $\frac{dy}{dx}$ $\left(\frac{dy}{dx}\right.$ ज्ञात करें। $\left.\right)$

1. $2x + 3y = \sin x$

2. $xy + y^2 = \tan x + y$

3. $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

4. $y = \sin^{-1}(2x.\sqrt{1-x^2})$

5. $y = \sin(\tan^{-1} e^{-x})$

6. $y = \log(\cos e^{x})$

7. $y = e^{x} + e^{x^2} + ... + e^{x^5}$

8. $y = (\log x)^{\cos x}$

9. $x^y + y^x = 1$

10. $y^x = x^y$

11. $(\cos y)^x = (\cos x)^y$

12. $x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

13. if $y = 5\cos x - 3\sin x$, then prove that $\frac{d^2y}{dx^2} + y = 0$

यदि $y = 5\cos x - 3\sin x$, तो सिद्ध करें $\frac{d^2y}{dx^2} + y = 0$

14. If $y = 3\sin x + \log x$ then find $\frac{d^2y}{dx^2}$

यदि $y = 3\sin x + \log x$ तो $\frac{d^2y}{dx^2}$ ज्ञात करें।

15. If $y = \tan^{-1}x$ then find $\frac{dy}{dx}$ at $x = 1$

यदि $y = \tan^{-1}x$ तो $x = 1$ पर $\frac{dy}{dx}$ ज्ञात करें।

16. Show that $f(x) = |x-2|$ is continuous but not differentiable at $x = 2$

सिद्ध कीजिए कि $x = 2$ पर $f(x) = |x-2|$ संतत है, परन्तु अवकलनीय नहीं है।

Solutions of short questions (लघु उत्तरीय प्रश्न का हल)

1. $2x + 3y = \sin x$

diff . both sides w.r.t x

$2 + 3\frac{dy}{dx} = \cos x$

$3\frac{dy}{dx} = \cos x - 2$

$\frac{dy}{dx} = \frac{\cos x - 2}{3}$

2. $xy + y^2 = \tan x + y$

diff . both sides w.r.t x

$x.\frac{dy}{dx} + y.1 + 2y\frac{dy}{dx} = \sec^2x + \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}.(x + 2y - 1) = \sec^2x - y$

$\Rightarrow \frac{dy}{dx} = \frac{\sec^2x - y}{x + 2y - 1}$

3. $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

put $x = \tan\theta \quad \therefore \quad \theta = \tan^{-1} x$

$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$

$y = \tan^{-1}(\tan 3\theta)$

$y = 3\theta$

$y = 3.\tan^{-1}x$

diff. both sides w.r. to x

$$\frac{dy}{dx} = \frac{3}{1+x^2}$$

4.

$y = \sin^{-1}(2x\sqrt{1-x^2})$

put $x = \sin\theta \quad \therefore \quad \theta = \sin^{-1} x$

$y = \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$y = \sin^{-1}(2\sin\theta.\cos\theta)$

$y = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\sin^{-1}x$

d.w.r. to x

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

5. $y = \sin(\tan^{-1} e^{-x})$

d.w.r. to x

$\frac{dy}{dx} = \frac{d\sin(\tan^{-1} e^{-x})}{dx}$

$= \frac{d\sin(\tan^{-1} e^{-x})}{d(\tan^{-1} e^{-x})} \times \frac{d\tan^{-1} e^{-x}}{d e^{-x}} \times \frac{d e^{-x}}{d(x)}$

$= \cos(\tan^{-1} e^{-x}) \times \frac{1}{1+(e^{-x})^2} \times e^{-x} \times (-1)$

$= \frac{-e^{-x}.\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}$

6. $y = \log(\cos e^x)$

d.w.r. to x

$\frac{dy}{dx} = \frac{d\log(\cos e^x)}{dx}$

$= \frac{d\log(\cos e^x)}{d(\cos e^x)} \times \frac{d\cos e^x}{de^x} \times \frac{de^x}{dx}$

$= \frac{1}{\cos e^x} \times (-\sin e^x) \times e^x$

$= -e^x.\tan(e^x)$

7. $y = e^x + e^{x^2} + ... + e^{x^5}$

$y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$

d.w.r. to x

$\frac{dy}{dx} = e^x + 2x.e^{x^2} + 3x^2.e^{x^3} + 4x^3e^{x^4} + 5x^4.e^{x^5}$

8. $y = (\log x)^{\cos x}$

taking log both sides

$\log y = \log(\log x)^{\cos x}$

$\log y = \cos x.\log(\log x)$

diff.both sides w.r.t x

$\frac{1}{y}\cdot\frac{dy}{dx} = \cos x.\frac{d\log(\log x)}{dx} + \log(\log x).\frac{d\cos x}{dx}$

$\Rightarrow \frac{dy}{dx} = y.\left[\cos x.\frac{1}{\log x} \times \frac{1}{x} + \log(\log x).(-\sin x)\right]$

$= (\log x)^{\cos x}.\left[\frac{\cos x}{x.\log x} - \sin x.\log(\log x)\right]$

9.

$x^y + y^x = 1$

d.w.r to x both sides

$\frac{d\,x^y}{dx} + \frac{d\,y^x}{dx} = \frac{d(1)}{dx}$

$\Rightarrow x^y.\left[\frac{y}{x}\cdot\frac{dx}{dx} + \log x.\frac{dy}{dx}\right] + y^x\left[\frac{x}{y}\cdot\frac{dy}{dx} + \log y.\frac{dx}{dx}\right] = 0$

$\Rightarrow \frac{y.x^y}{x} + x^y.\log x.\frac{dy}{dx} + \frac{x.y^x}{y}\cdot\frac{dy}{dx} + y^x\log y = 0$

$\Rightarrow \frac{dy}{dx}\left[x^y.\log x + \frac{x.y^x}{y}\right] = -\left[y^x.\log y + \frac{y.x^y}{x}\right]$

$\Rightarrow \frac{dy}{dx}\left[x^y.\log x + x.y^{x-1}\right] = -\left[y^x.\log y + y.x^{y-1}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{-[y^x.\log y + yx^{y-1}]}{[x^y.\log x + xy^{x-1}]}$

10.

$y^x = x^y$

taking log both sides

$\Rightarrow \log y^x = \log x^y$

$\Rightarrow x.\log y = y.\log x$

diff. both sides w.r.t x

$\Rightarrow x.\frac{1}{y}.\frac{dy}{dx} + \log y = y.\frac{1}{x} + \log x.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \frac{y}{x} - \log y$

$\Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x} - \log y\right)}{\left(\frac{x}{y} - \log x\right)} = \frac{y}{x}\left(\frac{y - x\log y}{x - y\log x}\right)$

11. $(\cos x)^y = (\cos y)^x$

taking log both sides

दोनों पक्षों में log लेने पर

$y. \log(\cos x) = x.\log(\cos y)$

diff. both sides w.r.t x

$$y.\frac{1}{\cos x}(-\sin x) + \log(\cos x)\frac{dy}{dx} = x.\left(\frac{-\sin y}{\cos y}\right)\frac{dy}{dx} + \log(\cos y)$$

$$\Rightarrow -y\tan x + \log(\cos x)\frac{dy}{dx} = -x\tan y\frac{dy}{dx} + \log(\cos y)$$

$$\Rightarrow \frac{dy}{dx}[\log(\cos x) + x\tan y] = \log(\cos y) + y\tan x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\log(\cos y) + y\tan x}{\log(\cos x) + x\tan y}$$

12. $x = a(\theta - \sin\theta)$ $\quad y = a(1 + \cos\theta)$

d.w.r. to θ $\quad$ d.w.r. to θ

$$\frac{dx}{d\theta} = a(1 - \cos\theta),\ \frac{dy}{d\theta} = -a\sin\theta$$

$$\text{Now } \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$$

$$= \frac{-2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$

$$= -\cot\frac{\theta}{2}$$

13.

$y = 5\cos x - 3\sin x$ —— (1)

d.w.r. to x

$\frac{dy}{dx} = -5\sin x - 3\cos x$ —— (2)

again d.w.r. to x

$\frac{d^2y}{dx^2} = -5\cos x + 3\sin x$ —— (3)

Adding (1) and (3) we get

(1) तथा (3) को जोड़ने पर

$$\frac{d^2y}{dx^2} + y = 0$$

14. $y = 3\sin x + \log x$ —— (1)

d.w.r. to x

$$\frac{dy}{dx} = 3\cos x + \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = -3\sin x - \frac{1}{x^2}$$

15. $y = \tan^{-1}x$ —— (1)

d.w.r. to x

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

at x = 1 (x = 1 पर)

$$\left.\frac{dy}{dx}\right]_{x=1} = \frac{1}{1+(1)^2} = \frac{1}{1+1} = \frac{1}{2}$$

16. Given

$f(x) = |x-2|$ —— (1)

at x = 2

$$\lim_{x\to2^+} f(x) = \lim_{h\to0} f(2+h) = \lim_{h\to0} |2+h-2| = \lim_{h\to0} h = 0$$

$$\lim_{x\to2^-} f(x) = \lim_{h\to0} f(2-h) = \lim_{h\to0} |2-h-2| = \lim_{h\to0} h = 0$$

and

$f(2) = |2-2| = 0$

$\therefore\ f(2) = \lim_{x\to2^+} f(x) = \lim_{x\to2^-} f(x)$

$\therefore$ f(x) is continuous at x = 2 **Proved**

x = 2 पर f(x) संतत है।

Again :-

$$Rf'(2) = \lim_{h\to0}\frac{f(2+h) - f(2)}{h} = \lim_{h\to0}\frac{|2+h-2| - |2-2|}{h} = \lim_{h\to0}\frac{h-0}{h} = 1$$

$$Lf'(2) = \lim_{h\to0}\frac{f(2-h) - f(2)}{-h} = \lim_{h\to0}\frac{|2-h-2| - |2-2|}{-h} = \lim_{h\to0}\frac{h}{-h} = -1$$

$\therefore\ Rf'(2) \neq Lf'(2)$

$\therefore$ f(x) is not differentiable at x = 2

x = 2 पर f(x) अवकलनीय है। **Proved**

Chapter-6	Application of Derivatives अवकलज के अनुप्रयोग

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 $f(x) = \sin x$ is increasing in

$f(x) = \sin x$ किस अन्तराल में वर्धमान है –

(a) $\left(\frac{\pi}{2}, \pi\right)$ (b) $\left(\pi, \frac{3\pi}{2}\right)$

(c) $(0, \pi)$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q 2 The function $f(x) = x^3 - 6x^2 + 9x + 3$ is decreasing for

फलन $f(x) = x^3 - 6x^2 + 9x + 3$ के ह्रासमान के लिए

(a) $|x| < 3$ (b) $|x| > 3$

(c) $-3 < x < 3$ (d) None of these/ इनमें से कोई नहीं।

Q 3 The least value of k for which $f(x) = x^2 + kx + 1$ is increasing on $(1,2)$ is-

k के किस मान के लिए $f(x) = x^2 + kx + 1$, अन्तराल $(1,2)$ पर वर्धमान है ?

(a) -2 (b) -1

(c) 1 (d) 2

Q 4 The least value of $f(x) = e^x + e^{-x}$ is

$f(x) = e^x + e^{-x}$ का न्यूनत्तम मान है-

(a) -2 (b) 0

(c) 2 (d) None of these/ इनमें से कोई नहीं

Q 5 The maximum value of $f(x) = (x-2)(x-3)^2$ is-

$f(x) = (x-2)(x-3)^2$ का महत्तम मान है-

(a) $\frac{7}{3}$ (b) 3

(c) $\frac{4}{27}$ (d) 0

Q 6 Tangent is parallel to x - axis then slope-

स्पर्श रेखा x-अक्ष के समान्तर है तो प्रवणता-

(a) -1 (b) 0

(c) ∞ (d) None /कोई नहीं

Q 7 Slope of tangent at $x = 2$, where curve $y = x^2 - 2x$

वक्र $y = x^2 - 2x$ में $x = 2$ पर स्पर्श रेखा का ढाल =

(a) 2 (b) 4

(c) -2 (d) 0

Q 8 Slope of line $4x - 2y + 8 = 0$ is-

रेखा $4x - 2y + 8 = 0$ का ढाल है-

(a) 4 (b) -2

(c) 8 (d) 2

Q 9 If curve $y = x^2$ then slope of normal at $x = 1$ is-

यदि वक्र $y = x^2$ है तो $x = 1$ पर अभिलम्ब का प्रवणता है–

(a) 2 (b) -2

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

Q 10 The slope at the point $(1, \sqrt{3})$ of the curve $x^2 + y^2 = 4$ is-

वक्र $x^2 + y^2 = 4$ के बिन्दू $(1, \sqrt{3})$ पर ढाल है –

(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $-\sqrt{3}$ (d) 3

Q 11 If m_1 and m_2 are slope of tangents at any point on the curve.The tangents are perpendicular if

यदि m_1 और m_2 वक्र के किसी बिंदु पर दो स्पर्श रेखाओं की ढाल हैं–

स्पर्श रेखाएँ परस्पर लम्बवत है यदि –

(a) $m_1 = m_2$ (b) $m_1 \times m_2 = -1$

(c) $\frac{m_1}{m_2} = 2$ (d) None /कोई नहीं।

Q 12 Tangent and normal is

स्पर्श रेखा और अभिलम्ब है -

(a) **Parallel to each other at point on the curve** /वक्र के बिन्दु पर समान्तर

(b) **Perpendicular each other at point on the curve** / वक्र के बिन्दु पर लम्बवत

(c) **Does not intersect each other**/एक दूसरे को प्रतिछेद नहीं करते

(d) None of these / इनमें से कोई नहीं।

Q 13 The rate of change of the area of a circle with respect to its radius r , when $r = 6cm$.

वृत के क्षेत्रफल के परिर्वतन की दर त्रिज्या r के सापेक्ष क्या होगी , यदि $r = 6cm$.

(a) $12\ cm^2/cm$ (b) $\pi\ cm^2/cm$

(c) $12\pi\ cm^2/cm$ (d) $6\pi\ cm^2/cm$

Q 14 The normal at the point $(1,1)$ on the curve $2y + x^2 = 3$ is

वक्र $2y + x^2 = 3$ के बिन्दू $(1,1)$ पर अभिलम्ब है-

(a) $x + y = 0$ (b) $x - y = 0$

(c) $x + y + 1 = 0$ (d) $x - y + 1 = 0$

Q 15 The equation of normal to the curve $x^2 = 4y$ passing through $(1,2)$ is

वक्र $x^2 = 4y$ कें बिन्दु $(1,2)$ से गुजरने वाले अभिलम्ब का समीकरण–

(a) $x + y = 3$ (b) $x - y = 3$

(c) $x + y = 1$ (d) $x - y = 1$

Q 16 The line $y = x + 1$ is a tangent to the curve $y^2 = 4x$ at the point

रेखा $y = x + 1$ वक्र $y^2 = 4x$ के किस बिन्दू पर स्पर्श करती है |

(a) $(1,2)$ (b) $(2,1)$

(c) $(1,-2)$ (d) $(-1,2)$

Q 17 The approximate value of $\sqrt{37}$ is

$\sqrt{37}$ का सन्निकट मान है-

(a) 6.08 (b) 7.08

(c) 5.6 (d) 9.2

Q 18 Equation of the tangent to the curve $y = x^2$ at $x = 1$ is

वक्र $y = x^2$ के $x=1$ पर स्पर्श रेखा का समीकरण है–

(a) $2x + y - 1 = 0$ (b) $2x - y - 1 = 0$

(c) $2x - y + 1 = 0$ (d) None of these / इनमें से कोई नहीं।

Q 19 If $f'(x) > 0$ for all $x \in]a,b[$ then $f(x)$ is -

यदि $f'(x) > 0$ सभी $x \in]a,b[$ तो $f(x)$ होगा

(a) **increasing** / वर्धमान

(b) **decreasing** / ह्ासमान

(c) **both increasing and decreasing** / वर्धमान और ह्ासमान दोनों

(d) **None of these** / इनमें से कोई नहीं

Q 20 The approximate value of $\sqrt{26}$ is

$\sqrt{26}$ का सन्निकट मान है -

(a) 6 (b) 5.1

(c) 5.8 (d) 5.9

Very Short Questions (2 Marks)

Q 1 In the curve $y^2 = 4x$ obtain the slope of the curve at the point where $y = 2$

वक्र $y^2 = 4x$ में वक्र के उस बिन्दू का प्रवणता निकाले जहाँ $y = 2$

Q 2 Find the point on the curve $y = x^3 - 2x^2 + x - 2$ where the tangents are parallel to the x - axis

वक्र $y = x^3 - 2x^2 + x - 2$ पर उन बिन्दूओं को मालूम करें जिन पर की स्पर्श रेखाएँ x-अक्ष के समान्तर हों|

Q 3 Find the rate of change of the area of a circle with respect to its radius r when $r = 3cm$

त्रिज्या r के सापेक्ष वृत के क्षेत्रफल के परिर्वतन की दर ज्ञात करें जब $r = 3cm$

Q 4 Examine whether the function $f(x) = x^2 - 4x + 3$ is increasing or decreasing at $x = 1$?

बताएँ कि फलन $f(x) = x^2 - 4x + 3$, $x = 1$ पर वर्धमान है या ह्ासमान है ?

Q 5 Find the approximate change in the volume v of a cube of side x caused by increasing the side by 1 %

यदि घन की भुजा x ,1% वृद्धि होती है तो आयतन v के सन्निकट परिर्वतन ज्ञात करें ?

Q 6 **Find the maximum value of $f(x) = x^2 - 4x + 2$**

फलन $f(x) = x^2 - 4x + 2$ के उच्चिष्ठ मान ज्ञात करें ?

Q 7 **Show that the function given by $f(x) = 3x + 17$ is strictly increasing on R**

सिद्ध कीजिए R पर $f(x) = 3x + 17$ से प्रदत फलन निरंतर वर्धमान है |

Q 8 **Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$?**

वक्र $x = a\cos^3\theta$, $y = a\sin^3\theta$ के $\theta = \frac{\pi}{4}$ पर अभिलम्ब की प्रवणता ज्ञात कीजिए ?

Short Questions (3 Marks)

Q 1 **The radius of a circle is increasing at the rate of 0.7cm/s . What is the rate of increase of its circumference ?**

एक वृत की त्रिज्या 0.7cm/s की दर से बढ़ रही है|इसकी परिधि की वृद्धि की दर क्या है?

Q 2 **A balloon which remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the radius is 10 cm**

एक गुब्बारा जो सदैव गोलाकार रहता है, की त्रिज्या परिर्वतनशील है | त्रिज्या के सापेक्ष आयतन के परिर्वतन की दर ज्ञात कीजिए जब त्रिज्या 10 cm है |

Q 3 **Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is**

अंतराल ज्ञात कीजिए जिनमें $f(x) = 2x^2 - 3x$ से प्रदत्त फलन f -

(a) strictly increasing / निरंतर वर्धमान

(b) strictly decreasing / निरंतर ह्रसमान

Q 4 **Prove that the logarithmic function is strictly increasing on $(0, \infty)$**

सिद्ध कीजिए कि लघुगणकीय फलन $(0, \infty)$ में निरंतर वर्धमान फलन है |

Q 5 **Find the equation of all lines having slope -1 that are tangent to the curve $y = \frac{1}{x-1}, x \neq 1$**

प्रवणता -1 वाली सभी रेखाओं का समीकरण ज्ञात कीजिए जो वक्र $y = \frac{1}{x-1}, x \neq 1$ को स्पर्श करती है |

Q 6 **Find the equation of the normal at the point (am^2, am^3) for the curve $ay^2 = x^3$?**

वक्र $ay^2 = x^3$ के बिन्दु (am^2, am^3) पर अभिलम्ब का समीकरण ज्ञात कीजिए ?

Q 7 **Find the maximum and minimum values of $f(x) = 9x^2 + 12x + 2$?**

फलन $f(x) = 9x^2 + 12x + 2$ का महत्तम और निम्नत्तम मान ज्ञात करें ?

Q 8 **Find the maximum profit that a company can make if the profit function is given by $p(x) = 41 - 72x - 18x^2$?**

यदि लाभ फलन $p(x) = 41 - 72x - 18x^2$ से प्रदत है तो किसी कंपनी द्वारा अर्जित उच्चत्तम लाभ ज्ञात कीजिए ?

Long Questions (5 Marks)

Q 1 **Find two numbers whose sum is 24 and whose product is large as possible ?**

ऐसी दो संख्याएँ ज्ञात कीजिए जिनका योग 24 है और जिनका गुणनफल उच्चत्तम हों ?

Q 2 **Show that all the rectangles inscribed in a given fixed circle , the square has the maximum area.**

सिद्ध कीजिए कि एक दिए गए वृत के अंतर्गत सभी आयतों में वर्ग का क्षेत्रफल उच्चत्तम होता है|

Q 3 **Find the maximum and minimum values of $3x^4 - 8x^3 + 12x^2 - 48x + 25$ in $[0, 3]$.?**

अंतराल $[0, 3]$ पर $3x^4 - 8x^3 + 12x^2 - 48x + 25$ के उच्चत्तम मान और निम्नतम मान ज्ञात कीजिए ?

Q 4 **Prove that $y = \frac{4\sin\theta}{2 + \cos\theta} - \theta$, is increasing in $\left[0, \frac{\pi}{2}\right]$.**

सिद्ध कीजिए कि $\left[0, \frac{\pi}{2}\right]$ में, $y = \frac{4\sin\theta}{2 + \cos\theta} - \theta$ एक वर्धमान फलन है |

Q 5 **Find points on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are**

(a) parallel to x-axis **(b) parallel to y-axis**

वक्र $\frac{x^2}{9} + \frac{y^2}{16} = 1$ पर उन बिन्दुओं को ज्ञात कीजिए जिन पर स्पर्श रेखाएँ

(a) x-अक्ष के समांतर है

(b) y-अक्ष के समांतर है

Q 6 **Show that the closed right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.**

सिद्ध कीजिए कि प्रदत पृष्ठ एंव महत्तम आयतन के बेलन की ऊँच्चाई , आधार के व्यास के बराबर होती है|

Q 7 **Find the absolute maximum value and the absolute minimum value of**

$f(x) = \sin x + \cos x\ ,\ x \in [0, \pi]$?

अंतराल $x \in [0, \pi]$ में फलन $f(x) = \sin x + \cos x$ के निरपेक्ष महत्तम और निरपेक्ष निम्नतम मान ज्ञात करें ?

Answer : "Application of Derivatives Solution"

Solution of MCQ
91 - Marks)

1 - (d) 2 - (a) 3 - (a) 4 - (c) 5 - (c) 6 - (b) 7 - (a)
8 - (d) 9 - (d) 10 - (a) 11 - (b) 12 - (b) 13 - (c)
14 - (b) 15 - (a) 16 - (a) 17 - (a) 18 - (b) 19 - (b)
20 - (b)

Solutions of very short questions
(2- Marks)

1 Ans : - **Given** $y^2 = 4x$ ....(1)

d.w.r.to x

$$2y\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{2}{y} \quad ....(2)$$

at y = 2

$$\text{slope} = \left.\frac{dy}{dx}\right]_{y=2} = \frac{2}{2} = 1$$

2 Ans:- **Given curve**

$$y = x^3 - 2x^2 + x - 2 \quad ....(1)$$

d.w.r.to x

$$\frac{dy}{dx} = 3x^2 - 4x + 1 \quad ....(2)$$

tangents are parallel to x - axis

$$\therefore \frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 - 4x + 1 = 0 \Rightarrow (3x - 1)(x - 1) = 0$$

$$\Rightarrow x = 1, \frac{1}{3}$$

From equation (1)

when $x = 1 \Rightarrow$

$$y = 1^3 - 2.1^2 + 1 - 2$$

$$y = 1 - 2 + 1 - 2 = -2$$

$$\therefore \text{Point} = (1, -2)$$

when $x = \frac{1}{3} \Rightarrow$

$$y = \left(\frac{1}{3}\right)^3 - 2.\left(\frac{1}{3}\right)^2 + \frac{1}{3} - 2$$

$$y = -\frac{50}{27}$$

$$\therefore \text{Point} = \left(\frac{1}{3}, -\frac{50}{27}\right)$$

3 Ans : -

Let

A = Area of circle (वृत का क्षेत्रफल)

r = radius of circle (वृत की त्रिज्या)

we know

$$A = \pi r^2$$

d.w.r. to r

$$\frac{dA}{dr} = 2\pi r$$

at r = 3cm

$$\left.\frac{dA}{dr}\right]_{r=3cm} = 2\pi(3)$$

$$= 6\pi\ cm^2/cm$$

4 Ans : -

$$f(x) = x^2 - 4x + 3 \quad ....(1)$$

d.w.r. to x

$$f'(x) = 2x - 4 \quad ....(2)$$

For increasing or decreasing at x = 1

x=1 पर वर्धमान या ह्रासमान के लिए

$$f'(1) = 2.(1) - 4$$

$$= 2 - 4$$

$$= -2 < 0$$

So , f(x) is decreasing at x = 1

x=1 पर f(x) ह्रासमान है।

5 Ans : - Let, **v = volume of cube of side x**

(भुजा x वाले घन का आयतन)

$$\therefore v = x^3$$

d.w.r. to x

$$\frac{dv}{dx} = 3x^2$$

Now change in volume

(अब आयतन में परिर्वतन)

$$\Delta v = \frac{dv}{dx} \cdot \Delta x$$

$$= 3x^2 \times \Delta x$$

$$= 3x^2 \times (1\% \text{ of } x)$$

$$= 3x^2 \times \frac{1}{100} \times x$$

$$= 0 \cdot 03x^3$$

6 Ans : -

$$f(x) = x^2 - 4x + 2 \text{ ....}(1)$$

d.w.r. to x

$$f'(x) = 2x - 4$$

$$f''(x) = 2 \text{ ....}(2)$$

for critical point,

$$f'(x) = 0$$

$$\Rightarrow 2x - 4 = 0 \Rightarrow 2x = 4$$

$$\therefore x = 2$$

Now from equation(2) $\Rightarrow x = 2$

$\Rightarrow f''(2) = 2 = +ve$ Maximum

$\therefore$ equation(1) $\Rightarrow x = 2$

$$\Rightarrow f(2) = 2^2 - 4(2) + 2$$

$$= 4 - 8 + 2$$

$$= -2$$

7 Ans : -

Given $f(x) = 3x + 17$

Let $x_1, x_2 \in R$

$$x_1 < x_2$$

$$\Rightarrow 3x_1 < 3x_2$$

$$\Rightarrow 3x_1 + 17 < 3x_2 + 17$$

$$\Rightarrow f(x_1) < f(x_2)$$

Hence , f(x) is strictly increasing on R

अतः R पर f(x) निरंतर वर्धमान है।

8 Ans : -

Given $x = a\cos^3\theta$

d.w.r.to θ

$$\frac{dx}{d\theta} = 3a\cos^2\theta(-\sin\theta)$$

And $y = a\sin^3\theta$

d.w.r.to θ

$$\frac{dy}{d\theta} = 3a\sin^2\theta(\cos\theta)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta(\cos\theta)}{3a\cos^2\theta(-\sin\theta)}$$

$$= -\tan\theta$$

at $\theta = \frac{\pi}{4}$

slope of normal = अभिलंब की प्रवणता $= \left.\frac{-1}{\frac{dy}{dx}}\right]_{\theta=\frac{\pi}{4}}$

$$= \frac{-1}{-\tan\frac{\pi}{4}}$$

$$= 1$$

Solutions of short questions
(3- Marks)

1 Ans : -

Let r = radius of circle (वृत क त्रिज्या)

c = circumference of circle (वृत की परिधि)

we know

$$c = 2\pi r$$

d.w.r. to t

$$\frac{dc}{dt} = 2\pi\frac{dr}{dt} \text{ ....}(1)$$

given (दिया है)

rate of increase of radius $= \frac{dr}{dt} = o \cdot 7$cm/s

$\therefore$ equation(1) $\Rightarrow \frac{dc}{dt} = 2\pi(o \cdot 7)$

$$= 1 \cdot 4\pi \text{ cm/s}$$

2 Ans : -

Let r = radius of balloon (गुब्बारे की त्रिज्या)

v = volume of balloon (गुब्बारे की आयतन)

we know

$$v = \frac{4}{3}\pi r^3$$

d.w.r. to r

$$\frac{dv}{dr} = 4\pi r^2$$

at r = 10cm $\Rightarrow$

$$\left.\frac{dv}{dr}\right]_{r=10cm} = 4\pi(10)^2$$

$$= 400\pi \text{ cm}^3/\text{cm}$$

3 Ans : -

Given $f(x) = 2x^2 - 3x$

d.w.r. to x

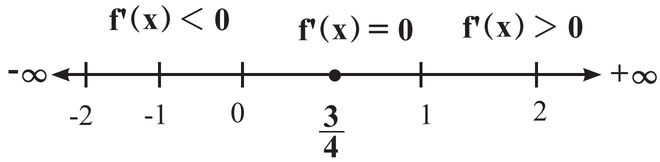
$$f'(x) = 4x - 3$$

for critical points –

$$f'(x) = 0$$

$$\Rightarrow 4x - 3 = 0 \quad \Rightarrow 4x = 3$$

$$\Rightarrow x = \frac{3}{4}$$



Clearly $f(x)$ is strictly increasing on $\left(\frac{3}{4}, \infty\right)$

$\left(\frac{3}{4}, \infty\right)$ अन्तराल पर f(x) निरंतर वर्धमान है।

and $f(x)$ is strictly decreasing on $\left(-\infty, \frac{3}{4}\right)$

, $\left(-\infty, \frac{3}{4}\right)$ अन्तराल पर f(x) निरंतर ह्रासमान है।

4 Ans : -

Let $f(x) = \log x \Rightarrow f'(x) = \frac{1}{x}$

when $x \in (0, \infty)$, $f'(x) > 0$

$\therefore$ $f(x)$ is strictly increasing in $(0, \infty)$

$(0, \infty)$ पर f(x) निरंतर वर्धमान है।

5 Ans : -

Given curve $y = \frac{1}{x-1}$, $x \neq 1$ ....(1)

d.w.r. to x

$$\frac{dy}{dx} = \frac{-1}{(x-1)^2}$$

$\Rightarrow -1 = \frac{-1}{(x-1)^2}$ $\left\{\because \frac{dy}{dx} = \text{slope} = -1\right\}$

$\Rightarrow (x-1)^2 = 1$

$\Rightarrow x^2 - 2x + 1 - 1 = 0$

$\Rightarrow x(x-2) = 0$

$\Rightarrow x = 0, 2$

When $x = 0 \Rightarrow$

from equation (1) $\Rightarrow$

$y = \frac{1}{0-1}$

$y = -1$

point $= (x, y)$

$= (0, -1)$

equation of tangent at $(0, -1)$

$y - y_1 = m(x - x_1)$

$y + 1 = -1(x - 0)$

$x + y + 1 = 0$

When $x = 2 \Rightarrow$

from equation (1) $\Rightarrow$

$y = \frac{1}{2-1}$

$y = 1$

point $= (x, y)$

$= (2, 1)$

equation of tangent at $(2, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - 2)$

$y - 1 = -x + 2$

$x + y - 3 = 0$

6 Ans.

Given curve (दिया गया वक्र)

$ay^2 = x^3$ ....(1)

d.w.r. to x

$$2ay\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

at (am^2, am^3)

slope of tangent (स्पर्श रेखा का ढाल) $= \left.\frac{dy}{dx}\right]_{(am^2, am^3)}$

$$= \frac{3(am^2)^2}{2a(am^3)}$$

$$= \frac{3m}{2}$$

Equation of normal at (am^2, am^3)

(am^2, am^3) पर अभिलंब का समीकरण

$$y - am^3 = \frac{-1}{\frac{3m}{2}}(x - am^2)$$

$$\Rightarrow y - am^3 = \frac{-2}{3m}(x - am^2)$$

$\Rightarrow 3my - 3am^4 = -2x + 2am^2$

$\Rightarrow 2x + 3my - 3am^4 - 2am^2 = 0$

7 Ans : -

Given function (दिया गया फलन)

$f(x) = 9x^2 + 12x + 2$ ....(1)

$= 9x^2 + 6x + 6x + 4 - 2$

$= (3x)^2 + 2 \cdot 3x \cdot 2 + (2)^2 - 2$

$f(x) = (3x+2)^2 - 2$ ....(2)

clearly,

$(3x+2)^2 \geq 0$

$(3x+2)^2 - 2 \geq -2$

For minimum (न्यूनत्तम के लिए)

$3x + 2 = 0$

$\Rightarrow x = -\frac{2}{3}$

$\therefore$ **minimum value of f** (f का न्यूनतम मान)

$f\left(-\frac{2}{3}\right) = \left(3 \times -\frac{2}{3} + 2\right)^2 - 2$

$= (-2+2)^2 - 2$

$= 0^2 - 2$

$= -2$

$\therefore f(x) \geq -2 \ \forall\ x$

$\therefore$ **f(x) has no maximum value.**
f(x) का महत्तम मान नहीं है।

8 Ans : -

Given

$p(x) = 41 - 72x - 18x^2$ ....(1)

$\frac{dp}{dx} = -72 - 36x$ ....(2)

$\frac{d^2p}{dx^2} = -36$ ....(3)

For critical point –

$\frac{dp}{dx} = 0$

$\Rightarrow -72 - 36x = 0$

$\Rightarrow 36x = -72$

$\Rightarrow x = -2$

equation(3) $\Rightarrow \left.\frac{d^2p}{dx^2}\right]_{x=-2} = -36 = -ve$

Maximum value at x = -2
x= -2 पर महत्तम मान है।

equation(1) $\Rightarrow p(-2) = 41 - 72(-2) - 18(-2)^2$

$= 41 + 144 - 72$

$= 113$

Solutions of long questions
(5- Marks)

1 Ans : -

Let one number = x

other number = 24 - x

Function (फलन)

y = product of two numbers
(दो संख्याओं का गुणा)

$y = x \cdot (24 - x)$ ....(1)

$\frac{dy}{dx} = 24 - 2x$ ....(2)

$\frac{d^2y}{dx^2} = -2$ ......(3)

For critical point

$\frac{dy}{dx} = 0$

$\Rightarrow 24 - 2x = 0$

$\Rightarrow x = 12$

eqnuation(3) $\Rightarrow \left.\frac{d^2y}{dx^2}\right]_{x=12} = -2(-ve)$

$\therefore$ **y is maximum at x = 12**

clearly

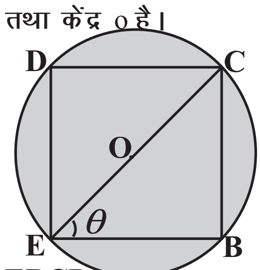
one number = x = 12

other number = 24 - x = 24 - 12 = 12

$\therefore$ **The numbers = 12 and 12**

2 Ans ;

Let EBCD be a rectangle inscribed in a circle of radius r and centre o.
(माना EBCD वृत के अन्तर्गत एक आयत है और वृत की त्रिज्या r तथा केंद्र o है।



Let $\angle CEB = \theta$

Then

EC = 2r , EB = 2rcosθ

and BC = 2rsinθ

Let A = area of rectangle EBCD
आयत EBCD का क्षेत्रफल

$A = EB \times BC$

$A = 2r\cos\theta \times 2r\sin\theta$

$A = 4r^2\cos\theta \cdot \sin\theta$

$A = 2r^2\sin 2\theta$ ....(1) ($\because$ r = constant)

$\frac{dA}{d\theta} = 4r^2\cos 2\theta$ ....(2)

$\frac{d^2A}{d\theta^2} = -8r^2\sin 2\theta$ ....(3)

For critical points $\Rightarrow \frac{dA}{d\theta} = 0$

$\Rightarrow 4r^2\cos 2\theta = 0$

$\Rightarrow \cos 2\theta = 0$

$\Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$

equation(3) $\Rightarrow \left.\frac{d^2A}{d\theta^2}\right]_{\theta=\frac{\pi}{4}} = -8r^2(-ve)$ **[maximum]**

$\therefore EB = 2r\cos\theta = 2r \cdot \cos\frac{\pi}{4} = 2r \times \frac{1}{\sqrt{2}} = \sqrt{2}\,r$

$$BC = 2r\sin\theta = 2r \cdot \sin\frac{\pi}{4} = 2r \times \frac{1}{\sqrt{2}} = \sqrt{2}\,r$$

$\therefore$ EB = BC

$\therefore$ EBCD is a square Proved

3 Ans : -

Given

$f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ ....(1)

d.w.r. to x

$f'(x) = 12x^3 - 24x^2 + 24x - 48$ ....(2)

$f''(x) = 36x^2 - 48x + 24$ ....(3)

For critical points –

$f'(x) = 0$

$\Rightarrow 12x^3 - 24x^2 + 24x - 48 = 0$

$\Rightarrow 12(x^3 - 2x^2 + 2x - 4) = 0$

$\Rightarrow 12[x^2(x-2) + 2(x-2)] = 0$

$\Rightarrow (x^2+2)(x-2) = 0$

$x^2 + 2 = 0$

$x^2 = -2$

$x = \sqrt{-2} \notin [0, 3]$

or

$x - 2 = 0 \in [0, 3]$

$x = 2$

$\therefore x = 2$

equation(3) $\Rightarrow f''(2) = 36(2)^2 - 48(2) + 24$

$= 144 - 96 + 24$

$= 72$ (+ve)

So, x = 2 is a point of local minima

Now equation(1) $\Rightarrow f(2) = -39$

$f(0) = 25$

$f(3) = 16$

$\therefore$ Maximum value of $f(x) = 25$ at $x = 0 \in [0, 3]$

$\therefore$ Minimum value of $f(x) = -39$ at $x = 2 \in [0, 3]$

4 Ans ; -

Given

$$y = \frac{4\sin\theta}{2+\cos\theta} - \theta \quad ....(1)$$

d.w.r. to θ

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left[\frac{4\sin\theta}{2+\cos\theta}\right] - \frac{d\theta}{d\theta}$$

$$= \frac{(2+\cos\theta)\frac{d}{d\theta}(4\sin\theta) - 4\sin\theta\frac{d(2+\cos\theta)}{d\theta}}{(2+\cos\theta)^2} - 1$$

$$= \frac{(2+\cos\theta)\cdot 4\cos\theta - 4\sin\theta\cdot(0-\sin\theta)}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4}{(2+\cos\theta)^2} - 1$$

$$= \frac{8\cos\theta + 4 - 4 - \cos^2\theta - 4\cos\theta}{(2+\cos\theta)^2}$$

$$= \frac{4\cos\theta - \cos^2\theta}{(2+\cos\theta)^2}$$

$$\frac{dy}{d\theta} = \frac{\cos\theta\cdot(4-\cos\theta)}{(2+\cos\theta)^2}$$

In $\left[0, \frac{\pi}{2}\right]$, $\cos\theta > 0 \Rightarrow -\cos\theta < 0 \Rightarrow 4 - \cos\theta > 0$

$\therefore \cos\theta(4-\cos\theta) \geq 0$ and $(2+\cos\theta)^2 > 0$

$$\Rightarrow \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2} \geq 0 \Rightarrow \frac{dy}{d\theta} \geq 0$$

So, y is increasing on $\left[0, \frac{\pi}{2}\right]$

5 Ans : -

Given

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \text{ ....(1)}$$

d.w.r. to x

$$\frac{2x}{9} + \frac{2y}{16}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2y}{16}\frac{dy}{dx} = -\frac{2x}{9}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{16x}{9y} \text{ ....(2)}$$

(a) tangent is parallel to x - axis

स्पर्श रेखा x-अक्ष के समान्तर है

$$\therefore \frac{dy}{dx} = 0 \Rightarrow -\frac{16x}{9y} = 0$$

$\therefore x = 0$

equation(1) $\Rightarrow 0 + \frac{y^2}{16} = 1$

$y^2 = 16$

$y = \pm 4$

$\therefore$ points (बिन्दुएँ) $= (x, y) = (0, \pm 4)$

(b) tangent is parallel to y - axis

स्पर्श रेखा y-अक्ष के समान्तर है

$$\frac{dx}{dy} = 0 \Rightarrow -\frac{9y}{16x} = 0$$

$\therefore y = 0$

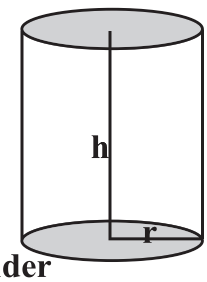
equation(1) $\Rightarrow \frac{x^2}{9} + 0 = 1 \Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

points (बिन्दुएँ) $= (x, y) = (\pm 3, 0)$

6 Ans ; -

Let



r = radius of cylinder
बेलन की त्रिज्या

h = height of cylinder
बेलन की ऊँचाई

s = surface area of cylinder
बेलन का पृष्ठीय क्षेत्रफल

v = volume of cylinder बेलन का आयतन

we know

$s = 2\pi r^2 + 2\pi rh$

$2\pi rh = s - 2\pi r^2 \quad \therefore h = \frac{s - 2\pi r^2}{2\pi r}$ ....(1)

and(और)

$v = \pi r^2 h$

$v = \pi r^2 \left(\frac{s - 2\pi r^2}{2\pi r} \right)$ {By (1)}

$v = \frac{sr}{2} - \pi r^3$ ....(2)

d.w.r. to r

$\frac{dv}{dr} = \frac{s}{2} - 3\pi r^2$ ....(3)

$\frac{d^2v}{dr^2} = -6\pi r$ ....(4)

For critical point –

$\frac{dv}{dr} = 0$

$\Rightarrow \frac{s}{2} - 3\pi r^2 = 0$

$\Rightarrow r^2 = \frac{s}{6\pi}$

$\Rightarrow r = \sqrt{\frac{s}{6\pi}}$

from eqution(4)

$\left. \frac{d^2v}{dr^2} \right]_{r=\sqrt{\frac{s}{6\pi}}} = -6\pi \sqrt{\frac{s}{6\pi}}$ (-ve)

$\therefore$ volume will be maximum

Now $r^2 = \frac{s}{6\pi}$

$s = 6\pi r^2$

from equaion(1)

$\Rightarrow h = \frac{6\pi r^2 - 2\pi r^2}{2\pi r}$

$\Rightarrow h = \frac{4\pi r^2}{2\pi r}$

$\Rightarrow h = 2r$

clearly height of cylinder is equal to diameter.
स्पष्ट है, बेलन की ऊँचाई व्यास के बराबर है।

7 Ans : -

$f(x) = \sin x + \cos x \quad , x \in [0, \pi]$ ....(1)

d.w.r. to x

$f'(x) = \cos x - \sin x$

For critical point –

$f'(x) = 0$

$\cos x - \sin x = 0$

$\tan x = 1 \quad \therefore x = \frac{\pi}{4} \in [0, \pi]$

Now equation(1) $\Rightarrow$

at $x = \frac{\pi}{4} \Rightarrow f\left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} + \cos\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

at $x = 0 \Rightarrow f(0) = \sin 0 + \cos 0 = 0 + 1 = 1$

at $x = \pi \Rightarrow f(\pi) = \sin\pi + \cos\pi = 0 - 1 = -1$

$\therefore$ absolute maximum value $= \sqrt{2}$ at $x = \frac{\pi}{4}$

निरपेक्ष महत्तम मान $= \sqrt{2}\ ;\ x = \frac{\pi}{4}$

$\therefore$ absolute minimum value $= -1$ at $x = \pi$

निरपेक्ष निम्नतम मान $= -1\ ;\ x = \pi$

Chapter : 7 INTEGRALS
समाकलन

MCQ :

1. $\int \sin 2x\, dx$ **is equal to** (बराबर हैं)

a) $2\cos 2x+c$ b) $\frac{1}{2}\cos 2x+c$
c) $-\frac{1}{2}\cos 2x+c$ d) $\cos 2x+c$

2. $\int \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$ **is equal to** (बराबर हैं)

a) $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+c$ b) $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}+c$

c) $\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+c$ d) $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+c$

3. $\int \cot x\, dx =$

a) $\log|\sin x|+c$ b) $\log|\cos x|+c$
c) $\log|\sec x|+c$ d) None of these (इनमें से कोई नहीं)

4. $\int \frac{dx}{\sin^2 x\cos^2 x} =$

a) $\tan x+\cot x+c$
b) $\tan x-\cot x+c$
c) $\tan x.\cot x+c$
d) $\tan x-\cot 2x+c$

5. $\int \frac{10x^9+10^x\log_e 10}{x^{10}+10^x}dx$ **equals** (बराबर है)

a) $10^x-x^{10}+c$ b) $10^x+x^{10}+c$
c) $(10^x-x^{10})^{-1}+c$ d) $\log(10^x+x^{10})+c$

6. $\int \frac{\sin^2 x-\cos^2 x}{\sin^2 x\cos^2 x}dx$ **is equal to** (बराबर है)

a) tanx+cotx+c
b) tanx+cosecx+c
c) -tanx +cotx+c
d) tanx+secx+c

7. $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx =$

a) $-\cot(ex^x)+c$ b) $\tan(xe^x)+c$
c) $\tan(e^x)+c$ d) $\cot(e^x)+c$

8. $\int \frac{dx}{x^2+2x+2}$ **equals** (बराबर है)

a) $x\tan^{-1}(x+1)+c$
b) $\tan^{-1}(x+1)+c$
c) $(x+1)\tan^{-1}x+c$
d) $\tan^{-1}x+c$

9. $\int \frac{dx}{\sqrt{9x-4x^2}}$ **equals** (बराबर है)

a) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+c$

b) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+c$

c) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+c$

d) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{9}\right)+c$

10. $\int \frac{x\,dx}{(x-1)(x-2)}$ **equals** (बराबर हैं)

a) $\log\left|\frac{(x-1)^2}{x-2}\right|+c$

b) $\log\left|\frac{(x-2)^2}{x-1}\right|+c$

c) $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+c$

d) $\log|(x-1)(x-2)|+c$

11. $\int \frac{dx}{x(x^2+1)}$ **equals** (बराबर है)

a) $\log|x|-\frac{1}{2}\log(x^2+1)+c$
b) $\log|x|+\frac{1}{2}\log(x^2+1)+c$
c) $-\log|x|+\frac{1}{2}\log(x^2+1)+c$
d) $\frac{1}{2}\log|x|+\log(x^2+1)+c$

12. $\int x^2 e^{x^3}dx$ **equals** (बराबर है)

a) $\frac{1}{3}e^{x^3}+c$
b) $\frac{1}{3}e^{x^2}+c$
c) $\frac{1}{2}e^{x^3}+c$
d) $\frac{1}{2}e^{x^2}+c$

13. $\int e^x\sec x(1+\tan x)dx =$

a) $e^x\cos x + c$ b) $e^x\sec x + c$
c) $e^x\sin x + c$ d) $e^x\tan x + c$

14. $\int \sqrt{1+x^2}\,dx$ **is equal to** (बराबर है)

a) $\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$

b) $\frac{2}{3}(1+x^2)^{\frac{3}{2}}+c$

c) $\frac{2}{3}x(1+x^2)^{\frac{3}{2}}+c$

d) $\frac{x^2}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+c$

15. $\int \frac{1}{x+\sqrt{x}}\,dx$ **is equal to** (बराबर है)

a) $\log|x|+\log(1+\sqrt{x})+c$
b) $2\log(1+\sqrt{x})+c$
c) $\log(1+\sqrt{x})+c$
d) $\log|x|+c$

16. $\int \frac{x^4+1}{x^2+1}\,dx =$

a) $\frac{x^3}{3}+x+\tan^{-1}x+c$
b) $\frac{x^3}{3}-x+\tan^{-1}x+c$
c) $\frac{x^3}{3}+x+2\tan^{-1}x+c$
d) $\frac{x^3}{3}-x+2\tan^{-1}x+c$

17. $\int \frac{dx}{e^x+e^{-x}}$ **is equal to** (बराबर है)

a) $\tan^{-1}(e^{x})+c$
b) $\tan^{-1}(e^{-x})+c$
c) $\log(e^x-e^{-x})+c$
d) $\log(e^x+e^{-x})+c$

18. $\int \frac{\cos 2x}{(\sin x+\cos x)^2}\,dx =$

a) $\frac{-1}{\sin x+\cos x}+c$

b) $\log|\sin x+\cos x|+c$

c) $\log|\sin x-\cos x|+c$

d) $\frac{1}{(\sin x+\cos x)^2}+c$

19 $\int \frac{e^x}{e^{2x}+1}\,dx$ **is equal to** (बराबर है)

a) $\log(e^x+e^{-x})+c$ b) $\tan^{-1}e^x+c$
c) $\log(e^{2x}+1)+c$ d) $\tan^{-1}(2e^x)+c$

20. $\int \cos\sqrt{x}\,dx$ **equals** (बराबर है)

a) $\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}+c$
b) $\sqrt{x}\sin\sqrt{x}-\cos\sqrt{x}+c$
c) $\frac{1}{2}\left[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right]+c$
d) $2\left[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right]+c$

21. $\int e^x(1+\tan x+\tan^2 x)\,dx =$

a) $e^x\cos x+c$ b) $e^x\sin x+c$
c) $e^x\tan x+c$ d) $e^x\sec x+c$

22. $\int \frac{dx}{1+x^2} =$

a) $\sin^{-1}x+c$ b) $\tan^{-1}x+c$
c) $\log(1+x^2)+c$ d) $\cot^{-1}x+c$

23. $\int \frac{\cos 2x+2\sin^2 x}{\cos^2 x}\,dx =$

a) $\tan x+c$ b) $\cot x+c$
c) $\sin x+c$ d) $\cos x+c$

24. $\int \frac{dx}{\sqrt{1-x^2}}$ **equals** (बराबर है)

a) $\cos^{-1}x+c$
b) $-\sin^{-1}x+c$
c) $\sin^{-1}x+c$
d) $\tan^{-1}x+c$

25. $\int e^x(\sin x+\cos x)\,dx =$

a) $e^x\sin x+c$ b) $e^x\cos x+c$
c) $e^x\tan x+c$ d) $\frac{e^x}{\sin x}+C$

26. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\,dx =$

a) $\cot x+x+c$ b) $\cot x-x+c$
c) $\tan x-x+c$ d) $\tan x+c$

27. $\int \frac{2}{\sqrt{1-4x^2}}\,dx =$

a) $\tan^{-1}(2x)+c$ b) $\cot^{-1}(2x)+c$
c) $\cos^{-1}(2x)+c$ d) $\sin^{-1}(2x)+c$

28. $\int_{-\pi/2}^{\pi/2} \sin^9 x\,dx =$

a) -1 b) 0
c) 1 c) None of these/ (इनमें से कोई नहीं)

29. $\int_0^p \frac{\sqrt{x}}{\sqrt{x}+\sqrt{p-x}}\,dx =$

a) p b) p/2
c) 2p d) none of these/ (इनमें से कोई नहीं)

30. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals (बराबर है)

a) $\frac{\pi}{3}$ b) $\frac{2\pi}{3}$
c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$

31. $\int_0^{2/3} \frac{dx}{4+9x^2}$ equals (बराबर है)

a) $\frac{\pi}{6}$ b) $\frac{\pi}{12}$
c) $\frac{\pi}{24}$ d) $\frac{\pi}{4}$

32. $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}dx$, is equal to (बराबर है)

a) 6 b) 0
c) 3 d) 4

33. $\int_{-\pi/2}^{\pi/2} (x^3+x\cos x+\tan^5 x+1)\,dx =$

a) 0 b) 2
c) π d) 1

34. $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx =$

a) 2 b) $\frac{3}{4}$
c) 0 d) -2

35. $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx =$

a) 1 b) 0
c) -1 d) $\frac{\pi}{4}$

36. $\int_0^1 \sqrt{x(1-x)}\,dx =$

a) $\frac{\pi}{8}$ b) $\frac{\pi}{6}$
c) $\frac{\pi}{4}$ d) $\frac{\pi}{2}$

37. $\int_1^{\sqrt{3}} \frac{1}{1+x^2}\,dx =$

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$
c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$

38. $\int_0^1 e^x dx =$

a) e - 1 b) $\frac{e-1}{e}$
c) $\frac{e^2-1}{e}$ d) $\frac{e^2-1}{2}$

39. $\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} =$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$
c) $\frac{\pi}{2}$ d) None of these/ (इनमें से कोई नहीं)

40. $\int_0^1 \frac{x}{1+x}\,dx =$

a) 1-log2 b) 2
c) 1+log2 d) log2

ANSWER

1.	c	21.	c
2.	c	22.	b
3.	a	23.	a
4.	b	24.	c
5.	d	25.	a
6.	a	26.	c
7.	b	27.	d
8.	b	28.	b
9.	b	29.	b
10.	b	30.	d
11.	a	31.	c
12.	a	32.	d
13.	b	33.	c
14.	a	34.	c
15.	b	35.	b
16.	d	36.	a
17.	a	37.	d
18.	b	38.	a
19.	b	39.	a
20.	d	40.	a

INTEGRALS (02 MARKS)

EVALUATE (मान निकाले)

1) $\int \tan x \, dx$

2) $\int \sec x \, dx$

3) $\int \text{cosec}\, x \, dx$

4) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$

5) $\int \frac{x^3 - x^2 + x - 1}{x - 1} dx$

6) $\int (1 - x)\sqrt{x} \, dx$

7) $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

8) $\int \frac{\sin(\tan^{-1} x)}{1 + x^2} dx$

9) $\int \sin^3 x \cos^2 x \, dx$

10) $\int \frac{1}{1 + \tan x} dx$

11) $\int \frac{\sin x}{\sin(x + a)} dx$

12. $\int \frac{(\log x)^2}{x} dx$

13. $\int \frac{1}{x + x\log x} dx$

14. $\int (x^3 - 1)^{1/3} x^5 \, dx$

15. $\int \frac{e^{\tan^{-1} x}}{1 + x^2} dx$

16. $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$

17. $\int \frac{\sin^{-1} x}{\sqrt{1 - x^2}} dx$

18. $\int \sec^2(7 - 4x) \, dx$

19. $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$

20. $\int \frac{(x + 1)(x + \log x)^2}{x} dx$

21. $\int \frac{1 - \cos x}{1 + \cos x} dx$

22. $\int \frac{\sin^2 x}{1 + \cos x} dx$

23. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$

24. $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx$

25. $\int \frac{3x^2}{x^6 + 1} dx$

26. $\int \frac{x^2}{1 - x^6} dx$

27. $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$

28. $\int \frac{1}{x^2 - 9} dx$

29. $\int \frac{1}{e^x - 1} dx$

30. $\int \log x \, dx$

31. $\int x e^x \, dx$

32. $\int x \sin x \, dx$

33. $\int \tan^{-1} x \, dx$

34. $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

35. $\int \sqrt{4-x^2}\,dx$

36. $\int \sqrt{x^2+4x+1}\,dx$

37. $\int \frac{1}{x-x^2}\,dx$

38. $\int \sqrt{\frac{x}{1-x}}\,dx$

39. $\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}\,dx$

40. $\int \frac{dx}{1+e^x}$

DEFINITE INTEGRALS (2 Marks)

1. $\int_{-1}^{1} (x+1)\,dx$

2. $\int_{2}^{3} \frac{1}{x}\,dx$

3. $\int_{0}^{\pi/2} \cos 2x\,dx$

4. $\int_{4}^{5} e^x\,dx$

5. $\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$

6. $\int_{2}^{3} \frac{dx}{x^2-1}$

7. $\int_{0}^{\pi/2} \cos^2 x\,dx$

8. $\int_{2}^{3} \frac{x}{x^2+1}\,dx$

9. $\int_{0}^{1} \frac{\tan^{-1}x}{1+x^2}\,dx$

10. $\int_{-\pi/4}^{\pi/4} \sin^2 x\,dx$

11. $\int_{0}^{1} x(1-x)^n\,dx$

12. $\int_{-1}^{1} e^x\,dx$

INDEFINITE INTEGRATION (3 MARKS)

EVALUATE (मान निकाले)

1. $\int (5x+3)\sqrt{2x-1}\,dx$

2. $\int \frac{x(x+3)}{x+5}\,dx$

3. $\int \sin^4 x\,dx$

4. $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}\,dx$

5. $\int \tan^4 x\,dx$

6. $\int \tan^3 x\,dx$

7. $\int \frac{dx}{x^2-a^2}$

8. $\int \frac{dx}{a^2-x^2}$

9. $\int \frac{dx}{x^2+a^2}$

10. $\int \frac{dx}{\sqrt{x^2-a^2}}$

11. $\int \frac{dx}{\sqrt{a^2-x^2}}$

12. $\int \frac{dx}{\sqrt{x^2+a^2}}$

13. $\int \frac{dx}{x^2-6x+13}$

14. $\int \frac{x+3}{\sqrt{5-4x-x^2}}dx$

15. $\int \frac{x+2}{\sqrt{x^2-1}}dx$

16. $\int \frac{x}{(x-1)(x-2)(x-3)}dx$

17. $\int e^x \sin x\, dx$

18. $\int x \sin^{-1} x\, dx$

19. $\int \frac{(x^2+1)e^x}{(x+1)^2}dx$

20. $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

21. $\int \sqrt{x^2+2x+5}\,dx$

22. $\int \sqrt{1+3x-x^2}\,dx$

23. $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

DEFINITE INTEGRALS (03 MARKS)

1. $\int_0^1 \frac{dx}{(\sqrt{x+1}+\sqrt{x})}$

2. $\int_0^{\pi/2} \frac{dx}{1+\sin x}$

3. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

4. $\int_0^1 |x-1|dx$

5. $\int_0^{\pi/2} \log \sin x\, dx$

6. $\int_0^{\pi/4} \log(1+\tan x)\, dx$

7. $\int_0^{2\pi} |\cos x|dx$

8. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

9. $\int_0^{2\pi} \cos^5 x\, dx$

10. $\int_0^{\pi} \frac{x}{1+\sin x}dx$

11. $\int_0^{\pi/2} \frac{x}{\sin x+\cos x}dx$

12. $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}dx$

DEFINITE INTEGRALS AS THE LIMIT OF A SUM (05 MARKS)

EVALUATE THE FOLLOWING DEFINITE INTEGRALS AS THE LIMIT OF SUM

1. $\int_0^1 x^2 dx$

2. $\int_0^1 e^x dx$

3. $\int_0^3 x\, dx$

4. $\int_0^4 (x+e^{2x})dx$

SOLUTION OF INTEGRATION(2 MARKS)

1. Let $I = \int \tan x\, dx = \int \frac{\sin x}{\cos x}dx$

put $\cos x = t$ so that $-\sin x dx = dt$

$\therefore \sin x\, dx = -dt$

$\int \tan x dx = -\int \frac{dt}{t}$

$= -\log|t| + C$

$= -\log|\cos x| + c$

$= \log|\sec x| + c$

$\therefore \int \tan x\, dx = -\log|\cos x| + c$

$= \log|\sec x| + c$

2. $Let\ \ I = \int \sec x\, dx$

$= \int \frac{\sec x(\sec x+\tan x)}{\sec x+\tan x} dx$

put $\sec x + \tan x = t$

$\sec x(\sec x + \tan x)\, dx = dt$

$\therefore I = \int \frac{dt}{t} = \log|t| + c$

$= \log|\sec x + \tan x| + c$

3. Let $I = \int \mathrm{cosec}\, x\, dx$

$= \int \frac{\mathrm{cosec}x(\mathrm{cosec}x-\cot x)}{(\mathrm{cosec}x-\cot x)} dx$

Put $\mathrm{cosec}\, x - \cot x = t$

$\therefore \mathrm{cosec}\, x(\mathrm{cosec}\, x - \cot x)\, dx = dt$

$\Rightarrow I = \int \frac{dt}{t} = \log|t| + c$

$= \log|\mathrm{cosec}\, x - \cot x| + c.$

4. $Let\ I = \int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx = \int \left(x + \frac{1}{x} - 2\right) dx$

$= \int x\, dx + \int \frac{1}{x} dx - \int 2\, dx$

$= \frac{x^2}{2} + \log|x| - 2x + c$

5. $I = \int \frac{x^3-x^2+x-1}{x-1} dx$

$= \int \frac{x^2(x-1)+1(x-1)}{x-1} dx$

$= \int \frac{(x-1)(x^2+1)}{x-1} dx$

$= \int (x^2+1)dx = \frac{x^3}{3} + x + c$

6. $I = \int (1-x)\sqrt{x}\, dx = \int (\sqrt{x} - x\sqrt{x})\, dx$

$= \int x^{1/2} dx - \int x^{3/2} dx$

$= \frac{x^{1/2+1}}{\frac{1}{2}+1} - \frac{x^{3/2+1}}{\frac{3}{2}+1} + c$

$= \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + c$

7. $I = \int \frac{2-3\sin x}{\cos^2 x} dx = \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right) dx$

$= 2\int \sec^2 x\, dx - 3\int \sec x \,.\, \tan x\, dx$

$= 2\tan x - 3\sec x + c$

8. $I = \int \frac{\sin(\tan^{-1}x)}{1+x^2} dx$

Put $\tan^{-1}x = t$, $\therefore \frac{1}{1+x^2}dx = dt$

$\therefore I = \int \sin t\, dt = -\cos(t) + c$

$= -\cos(\tan^{-1}x) + c$

9. $I = \int \sin^3 x\cos^2 x\, dx = \int \sin^2 x\, \cos^2 x\, \sin x\, dx$

$= \int (1 - \cos^2 x)\cos^2 x(\sin x)\, dx;$

put $\cos x = t$

$\Rightarrow -\sin x dx = dt$

$\therefore I = -\int (1-t^2)t^2 dt = -\int (t^2 - t^4)\, dt$

$= -\left(\frac{t^3}{3} - \frac{t^5}{5}\right) + c$

$= -\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c.$

10. $I = \int \frac{1}{1+\tan x} dx = \int \frac{\cos x}{\cos x+\sin x} dx$

$= \frac{1}{2}\int \frac{2\cos x}{\cos x+\sin x} dx$

$= \frac{1}{2}\int \frac{\cos x+\sin x+\cos x-\sin x}{\cos x+\sin x} dx$

$= \frac{1}{2}\int 1dx + \frac{1}{2}\int \frac{\cos x - \sin x}{\cos x + \sin x} dx$

$= \frac{1}{2}x + \frac{1}{2}\log|\cos x + \sin x| + c$

11. $Let\ \ I = \int \frac{\sin x}{\sin(x+a)} dx$

put $(x + a) = t \Rightarrow dx = dt$

$\therefore I = \int \frac{\sin(t-a)}{\sin t} dt$

$= \int \frac{\sin t\cos a - \cos t\sin a}{\sin t} dt$

$= \cos a\int dt - \int \sin a \cot t\, dt$

$= \cos a\,(t) - \sin a\int \cot t\, dt$

$= \cos a\,(t) - \sin a \log|\sin t| + c$

$= \cos a\,(x + a) - \sin a \log|\sin(x + a)| + c$

$= x\cos a + a\cos a - \sin a \log|\sin(x + a)| + c$

12. $let\ I = \int \frac{(\log x)^2}{x} dx$

put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore I = \int t^2 dt = \frac{t^3}{3} + c$

$= \frac{(\log x)^3}{3} + c$

13. $I=\int \frac{1}{x+x\log x}dx=\int \frac{1}{x(1+\log x)}dx$

put $1+\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore I=\int \frac{dt}{t}=\log|t|+c$

$=\log|1+\log x|+c.$

14. $I=\int (x^3-1)^{1/3}x^5dx=\int (x^3-1)^{\frac{1}{3}}x^3x^2dx$

put $x^3-1=t \Rightarrow 3x^2dx=dt \Rightarrow x^2dx=\frac{1}{3}dt$

$\therefore I=\frac{1}{3}\int t^{1/3}(t+1)dt=\frac{1}{3}\int (t^{4/3}+t^{1/3})dt$

$=\frac{1}{3}\left[\frac{t^{4/3+1}}{\frac{4}{3}+1}\right]+\frac{1}{3}\left[\frac{t^{1/3+1}}{\frac{1}{3}+1}\right]+c$

$=\frac{1}{7}t^{7/3}+\frac{1}{4}t^{4/3}+c$

$=\frac{1}{7}(x^3-1)^{7/3}+\frac{1}{4}(x^3-1)^{4/3}+c$

15. $I=\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$

put $\tan^{-1}x=t \Rightarrow \frac{1}{1+x^2}dx=dt$

$\therefore I=\int e^t dt \quad = e^t+c$

$=e^{\tan^{-1}x}+c$

16. $I=\int \frac{e^{2x}-1}{e^{2x}+1}dx=\int \frac{e^{2x}-1}{e^x}/\frac{e^{2x}+1}{e^x}dx$

$=\int \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$

put $e^x+e^{-x}=t \Rightarrow (e^x-e^{-x})dx=dt$

$\therefore I=\int \frac{dt}{t}=\log|t|+c=\log|e^x+e^{-x}|+c$

17. $I=\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

put $\sin^{-1}x=t \Rightarrow \frac{1}{\sqrt{1-x^2}}dx=dt$

$\therefore I=\int t\,dt=\frac{t^2}{2}+c=\frac{(\sin^{-1}x)^2}{2}+c$

18. $I=\int sec^2(7-4x)dx$

put $7-4x=t \Rightarrow -4dx=dt$

$\therefore dx=-\frac{1}{4}dt$

$\therefore I=-\frac{1}{4}\int \sec^2 t\,dt=-\frac{1}{4}\tan t+c$

$=-\frac{1}{4}\tan(7-4x)+c$

19. $I=\int \frac{1}{\cos^2x(1-\tan x)^2}dx=\int \frac{\sec^2x}{(1-\tan x)^2}dx$

put $1-\tan x=t \Rightarrow -\sec^2xdx=dt$

$\Rightarrow \sec^2x\,dx=-dt$

$I=-\int \frac{dt}{t^2}=\frac{1}{t}+c=\frac{1}{(1-\tan x)}+c$

20.

$I=\int \frac{(x+1)(x+\log x)^2}{x}dx$

$=\int (\frac{x+1}{x})(x+\log x)^2dx$

$=\int (1+\frac{1}{x})(x+\log x)^2dx$

put $x+\log x=t \Rightarrow (1+\frac{1}{x})dx=dt$

$\therefore I=\int t^2dt=\frac{t^3}{3}+c=\frac{(x+\log x)^3}{3}+c$

21. $I=\int \frac{1-\cos x}{1+\cos x}dx=\int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$

$=\int \tan^2\frac{x}{2}dx=\int\left(\sec^2\frac{x}{2}-1\right)dx$

$=\frac{\tan\frac{x}{2}}{1/2}-x+c$

$=2\tan\frac{x}{2}-x+c$

22. $I=\int \frac{\sin^2x}{1+\cos x}dx=\int \frac{1-\cos^2x}{1+\cos x}dx$

$=\int \frac{(1+\cos x)(1-\cos x)}{1+\cos x}dx=\int (1-\cos x)dx$

$=x-\sin x+c$

23. $I=\int \frac{\cos 2x+2\sin^2x}{\cos^2x}dx$

$=\int \frac{1-2\sin^2x+2\sin^2x}{\cos^2x}dx=\int \frac{1}{\cos^2x}dx$

$=\int \sec^2x\,dx=\tan x+c$

24. $I=\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx=\int \frac{\cos^2x-\sin^2x}{(\cos x+\sin x)^2}dx$

$=\int \frac{(\cos x+\sin x)(\cos x-\sin x)}{(\cos x+\sin x)^2}dx$

$=\int \frac{\cos x-\sin x}{\cos x+\sin x}dx$

$=\int \frac{1-\tan x}{1+\tan x}dx$

$=\int \tan(\frac{\pi}{4}-x)dx$

$=\log\left|\cos(\frac{\pi}{4}-x)\right|+c$

or,

$I=\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx=\int \frac{\cos^2x-\sin^2x}{(\cos x+\sin x)^2}dx$

$=\int \frac{(\cos x+\sin x)(\cos x-\sin x)}{(\cos x+\sin x)^2}dx$

$=\int \frac{\cos x-\sin x}{\cos x+\sin x}dx$

put $\cos x+\sin x=t$

$\Rightarrow (\cos x-\sin x)dx=dt$

$\therefore I=\int \frac{dt}{t}=\log|t|+c$

$=\log|\cos x+\sin x|+c$

25. $I=\int \frac{3x^2}{x^6+1}dx = \int \frac{3x^2}{(x^3)^2+1}dx$

put $x^3 = t \quad \Rightarrow 3x^2 dx = dt$

$\therefore I=\int \frac{dt}{t^2+1} = \tan^{-1}(t)+c$

$= \tan^{-1}(x^3)+c$

26. $I=\int \frac{x^2}{1-x^6}dx = \int \frac{x^2}{1-(x^3)^2}dx$

put $x^3 = t \quad \Rightarrow 3x^2 dx = dt$

$x^2 dx = \frac{1}{3}dt$

$\therefore I=\frac{1}{3}\int \frac{dt}{1-t^2} = \frac{1}{3}\int \frac{dt}{(1)^2-(t)^2}$

$= \frac{1}{3}\times\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|+c$

$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right|+c$

27. $I=\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}dx$

put $\tan x = t \quad \Rightarrow \sec^2 x\, dx = dt$

$\therefore I=\int \frac{dt}{\sqrt{t^2+4}} = \int \frac{dt}{\sqrt{t^2+2^2}}$

$= \log\left|t+\sqrt{t^2+4}\right|+c$

$= \log\left|\tan x+\sqrt{\tan^2 x+4}\right|+c.$

28. $I=\int \frac{1}{x^2-9}dx = \int \frac{1}{x^2-3^2}dx$

$= \frac{1}{2\times 3}\log\left|\frac{x-3}{x+3}\right|+c$

$= \frac{1}{6}\log\left|\frac{x-3}{x+3}\right|+c.$

29. $I=\int \frac{1}{e^x-1}dx$

put $e^x-1=t \quad \Rightarrow e^x dx = dt$

$\therefore dx = \frac{1}{e^x}dt = \frac{1}{(t+1)}dt$

$\therefore I=\int \frac{1}{t(t+1)}dt = \int \frac{t+1-t}{t(t+1)}dt$

$= \int\left(\frac{1}{t}-\frac{1}{t+1}\right)dt = \log|t|-\log|t+1|+c$

$= \log\left|\frac{t}{t+1}\right|+c = \log\left|\frac{e^x-1}{e^x}\right|+c$

30. $I=\int \log x\, dx = \int \log x .1dx$

$= \log x\int 1dx - \int\left[\frac{d}{dx}(\log x)\int 1dx\right]dx$

$= \log x.x - \int \frac{1}{x}.x\, dx$

$= x\log x - \int 1\, dx = x\log x - x + c$

$= x(\log x - 1)+c$

31. $I=\int xe^x dx$

$= x\int e^x dx - \int\left[\frac{d}{dx}(x)\int e^x dx\right]dx$

$= xe^x - \int 1.e^x dx$

$= xe^x - e^x + c$

$= (x-1)e^x + c$

32. $I=\int x\sin x\, dx$

$= x\int \sin x dx - \int\left[\frac{d}{dx}(x)\int \sin x dx\right]dx$

$= x(-\cos x) - \int 1.(-\cos x)\, dx$

$= -x\cos x + \int \cos x\, dx$

$= -x\cos x + \sin x + c$

33. $I=\int \tan^{-1}x\, dx = \int (\tan^{-1}x.1)\, dx$

$= \tan^{-1}x.\int 1dx - \int\left[\frac{d}{dx}(\tan^{-1}x)\int 1dx\right]dx$

$= x\tan^{-1}x - \int \frac{x}{1+x^2}dx$

$= x\tan^{-1}x - \frac{1}{2}\int \frac{2x}{1+x^2}dx$

$= x\tan^{-1}x - \frac{1}{2}\log|1+x^2|+c$

34. $I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$

consider $f(x)=\frac{1}{x}$, then $f'(x) = -\frac{1}{x^2}$

thus the given integrand is of the form $e^x[f(x)+f'(x)]$.

$\therefore I=\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = e^x.\frac{1}{x}+c$

$\therefore I=\frac{e^x}{x}+c.$

35. $I=\int \sqrt{4-x^2}\, dx$

$= \int \sqrt{2^2-x^2}\, dx$

$= \frac{x}{2}\sqrt{2^2-x^2}+\frac{2^2}{2}\sin^{-1}\frac{x}{2}+c$

$= \frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+c$

36. $I=\int \sqrt{x^2+4x+1}\, dx$

$= \int \sqrt{x^2+4x+4-4+1}\, dx$

$= \int \sqrt{(x+2)^2-3}\, dx$

$= \int \sqrt{(x+2)^2-(\sqrt{3})^2}\, dx$

$= \frac{x+2}{2}\sqrt{(x+2)^2-(\sqrt{3})^2} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+c$

$= \frac{x+2}{2}\sqrt{x^2+4x+1} - \frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|+c$

37. $I=\int \frac{1}{x-x^2}dx=\int \frac{1}{x(1-x)}dx$

$=\int (\frac{1}{x}+\frac{1}{1-x})\,dx$

$=\log|x|-\log|1-x|+c$

$=\log\left|\frac{x}{1-x}\right|+c$

38. $I=\int \sqrt{\frac{x}{1-x}}dx$

put $x=\sin^2 t$

$dx=2\sin t\cos t\,dt$

$=\int \sqrt{\frac{\sin^2 t}{1-\sin^2 t}}\cdot 2\sin t\cos t\,dt$

$=\int \frac{\sin t}{\cos t}\cdot 2\sin t\cos t\,dt$

$=\int 2\sin^2 t\,dt$

$=\int (1-\cos 2t)\,dt=t-\frac{\sin 2t}{2}+c$

$=t-\frac{2\sin t.\cos t}{2}+c$

$=t-\sin t.\cos t+c$

$=\sin^{-1}\sqrt{x}-\sqrt{x}.\sqrt{1-x}+c$

39. $I=\int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}dx$

put $\sin^{-1}x=t$

$\therefore \frac{1}{\sqrt{1-x^2}}dx=dt$

$\therefore I=\int t^3\,dt=\frac{t^4}{4}+c$

$=\frac{(\sin^{-1}x)^4}{4}+c$

40. $I=\int \frac{1}{1+e^x}dx$

put $1+e^x=t \;\Rightarrow e^x dx=dt$

$\Rightarrow dx=\frac{1}{e^x}dt$

$\Rightarrow dx=\frac{1}{t-1}dt$

$\therefore I=\int \frac{1}{t(t-1)}dt=\int (\frac{1}{t-1}-\frac{1}{t})\,dt$

$=\log|t-1|-\log|t|+c$

$=\log\left|\frac{t-1}{t}\right|+c$

$=\log\left|\frac{e^x}{1+e^x}\right|+c$

DEFINITE INTEGRATION (2 MARKS)

1. $I=\int_{-1}^{1}(x+1)\,dx=\left[\frac{x^2}{2}+x\right]_{-1}^{1}$

$=\frac{1}{2}\left[(1)^2-(-1)^2\right]+\left[1-(-1)\right]$

$=\frac{1}{2}(1-1)+(1+1)$

$=\frac{1}{2}\times 0+2=2$

2. $I=\int_2^3 \frac{1}{x}dx=\left[\log x\right]_2^3$

$=(\log 3-\log 2)$

$=\log\frac{3}{2}$

3. $I=\int_0^{\pi/2}\cos x dx=[\sin x]_0^{\pi/2}$

$=\sin\frac{\pi}{2}-\sin 0=1-0=1$

4. $I=\int_4^5 e^x dx=[e^x]_4^5$

$=e^5-e^4$

$=e^4(e-1).$

5. $I=\int_0^1 \frac{dx}{\sqrt{1-x^2}}=[\sin^{-1}x]_0^1$

$=\sin^{-1}1-\sin^{-1}0$

$=\frac{\pi}{2}-0=\frac{\pi}{2}$

6. $I=\int_2^3 \frac{dx}{x^2-1}=\frac{1}{2}\left[\log\frac{x-1}{x+1}\right]_2^3$

$=\frac{1}{2}\left[\log(x-1)-\log(x+1)\right]_2^3$

$=\frac{1}{2}\left[\log 2-\log\frac{4}{3}\right]$

$=\frac{1}{2}\log\frac{\frac{2}{4}}{3}=\frac{1}{2}\log\frac{3}{2}$

7. $I=\int_0^{\pi/2}\cos^2 x dx$ -------- (i)

$=\int_0^{\pi/2}\cos^2(\frac{\pi}{2}-x)\,dx$

$=\int_0^{\pi/2}\sin^2 x dx$ ------ (ii)

adding (i) and (ii), we get

$2I=\int_0^{\pi/2}(\cos^2 x+\sin^2 x)\,dx$

$=\int_0^{\pi/2}1.dx=[x]_0^{\pi/2}$

$\therefore 2I=\frac{\pi}{2} \;\Rightarrow I=\frac{\pi}{4}$

8. $I=\int_2^3 \frac{x}{x^2+1}dx$

put $x^2+1=t \Rightarrow 2x\,dx=dt$

$\therefore xdx=\frac{1}{2}dt$

when x = 2 then t = 5 and

when x = 3 then t = 10

$I=\frac{1}{2}\int_5^{10}\frac{dt}{t}=\frac{1}{2}[\log t]_5^{10}$

$=\frac{1}{2}[\log 10-\log 5]$

$=\frac{1}{2}\log\frac{10}{5}=\frac{1}{2}\log 2$

9. $I=\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$

put $\tan^{-1}x=t$

$\Rightarrow \frac{1}{1+x^2}dx=dt$

when x = 0 then t = 0 and

when x = 1 then $t=\frac{\pi}{4}$

$\therefore I=\int_0^{\pi/4} t\,dt = \left[\frac{t^2}{2}\right]_0^{\pi/4}$

$=\frac{\frac{\pi^2}{16}-0}{2}=\frac{\pi^2}{32}$

10. $I=\int_{-\pi/4}^{\pi/4}\sin^2x\,dx=2\int_0^{\pi/4}\sin^2x\,dx$

$=2\int_0^{\pi/4}\frac{1-\cos 2x}{2}dx=\int_0^{\pi/4}(1-\cos 2x)\,dx$

$=\left[x-\frac{\sin 2x}{2}\right]_0^{\pi/4}=\frac{\pi}{4}-\frac{1}{2}\sin\frac{\pi}{2}$

$=\frac{\pi}{4}-\frac{1}{2}$

11. $I=\int_0^1 x(1-x)^n dx=\int_0^1(1-x).x^n dx$

$=\int_0^1(x^n-x^{n+1})dx=\left[\frac{x^n}{n+1}-\frac{x^{n+2}}{n+2}\right]_0^1$

$=\frac{1}{n+1}-\frac{1}{n+2}=\frac{n+2-n-1}{(n+1)(n+2)}=\frac{1}{n^2+3n+2}$

12. $I=\int_{-1}^1 e^x dx=[e^x]_{-1}^1$

$=e^1-e^{-1}=e-\frac{1}{e}$

INDEFINITE INTEGRATION (03 MARKS)

1. $I=\int(5x+3)\sqrt{2x-1}\,dx$

$=(5x+3)\int\sqrt{2x-1}\,dx-\int\left\{\frac{d}{dx}(5x+3)\int\sqrt{2x-1}\,dx\right\}dx$

$=(5x+3)\frac{(2x-1)^{3/2}}{\frac{3}{2}\times 2}-\int 5\left\{\frac{(2x-1)^{3/2}}{\frac{3}{2}.2}\right\}dx$

$=\frac{1}{3}(5x+3)(2x-1)^{3/2}-\frac{5}{3}\int(2x-1)^{3/2}dx$

$=\frac{1}{3}(5x+3)(2x-1)^{3/2}-\frac{5}{3}.\frac{(2x-1)^{5/2}}{\frac{5}{2}.2}+c$

$=\frac{1}{3}(5x+3)(2x-1)^{3/2}-\frac{1}{3}(2x-1)^{5/2}+c$

$=\frac{1}{3}(2x-1)^{3/2}[5x+3-(2x-1)]+c$

$=\frac{1}{3}(2x-1)^{3/2}(3x+4)+c$

2. $I=\int\frac{x(x+3)}{x+5}dx=\int\frac{x^2+3x}{x+5}dx$

$=\int\left[(x-2)+\frac{10}{x+5}\right]dx$

$=\frac{x^2}{2}-2x+10\log|x+5|+c$

3. $I=\int\sin^4x\,dx=\int(\sin^2x)^2dx$

$=\int\left(\frac{1-\cos 2x}{2}\right)^2dx$

$=\frac{1}{4}\int(1+\cos^2 2x-2\cos 2x)\,dx$

$=\frac{1}{4}\int\left(1+\frac{1+\cos 4x}{2}-2\cos 2x\right)dx$

$=\frac{1}{4}\int\frac{2+1+\cos 4x-4\cos 2x}{2}dx$

$=\frac{1}{8}\int(3+\cos 4x-4\cos 2x)\,dx$

$=\frac{3x}{8}+\frac{\sin 4x}{32}-\frac{\sin 2x}{4}+c$

4. $I=\int\frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$

$=\int\frac{(2\cos^2x-1)-(2\cos^2\alpha-1)}{\cos x-\cos\alpha}dx$

$=2\int\frac{\cos^2x-\cos^2\alpha}{\cos x-\cos\alpha}dx$

$=2\int\frac{(\cos x+\cos\alpha)(\cos x-\cos\alpha)}{(\cos x-\cos\alpha)}dx$

$=2\int(\cos x+\cos\alpha)\,dx$

$=2\sin x+2x\cos\alpha+c$

5. $I=\int \tan^4 x dx=\int (\tan^2 x . \tan^2 x) dx$

$=\int \tan^2 x(\sec^2 x-1) dx$

$=\int (\tan^2 x \sec^2 x-\tan^2 x) dx$

$=\int (\tan^2 x \sec^2 x) dx-\int \tan^2 x\, dx$

$=\int \tan^2 x . \sec^2 x dx-\int (\sec^2 x-1) dx$

$=\int \tan^2 x . \sec^2 x dx-\int \sec^2 x\, dx+\int 1\, dx$

put $\tan x=t$ in 1st integral,

$\Rightarrow \sec^2 x dx=dt$

$\therefore I=\int t^2 dt-\tan x+x$

$=\frac{t^3}{3}-\tan x+x+c$

$=\frac{\tan^3 x}{3}-\tan x+x+c$

6. $I=\int \tan^3 x\, dx=\int \tan x \tan^2 x\, dx$

$=\int \tan x(\sec^2 x-1) dx$

$\int \tan x \sec^2 x\, dx-\int \tan x\, dx$

put $\tan x=t$ in first integral,

$\Rightarrow \sec^2 x=\frac{dt}{dx} \Rightarrow \sec^2 x\, dx=dt$

$\therefore I=\int t dt-\log|\sec x|$

$=\frac{t^2}{2}-\log|\sec x|+c$

$=\frac{\tan^2 x}{2}-\log|\sec x|+c$

7. $I=\int \frac{dx}{x^2-a^2}=\int \frac{1}{(x+a)(x-a)} dx$

$=\frac{1}{2a}\int \frac{(x+a)-(x-a)}{(x+a)(x-a)} dx$

$=\frac{1}{2a}\int (\frac{1}{x-a}-\frac{1}{x+a}) dx$

$=\frac{1}{2a}[\log|x-a|-\log|x+a|]+c$

$=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$

8. $I=\int \frac{dx}{a^2-x^2}=\int \frac{1}{(a+x)(a-x)} dx$

$=\frac{1}{2a}\int \frac{(a+x)+(a-x)}{(a+x)(a-x)} dx$

$=\frac{1}{2a}\int (\frac{1}{a-x}+\frac{1}{a+x}) dx$

$=\frac{1}{2a}[-\log|a-x|+\log|a+x|]+c$

$=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

9. $I=\int \frac{dx}{x^2+a^2}$

put $x=a\tan\theta$ *then* $dx=a\sec^2\theta d\theta$

$\therefore I=\int \frac{a\sec^2\theta\, d\theta}{a^2+a^2\tan^2\theta}=\int \frac{a\sec^2\theta}{a^2\sec^2\theta}d\theta$

$=\frac{1}{a}\int d\theta=\frac{1}{a}\theta+c=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

10. $I=\int \frac{dx}{\sqrt{x^2-a^2}}$

put $x=a\sec\theta$

$\therefore dx=a\sec\theta.\tan\theta d\theta$

$\therefore I=\int \frac{a\sec\theta.\tan\theta d\theta}{\sqrt{a^2\sec^2\theta-a^2}}=\int \frac{a\sec\theta.\tan\theta}{a\tan\theta}d\theta$

$=\int \sec\theta d\theta \;=\log|\sec\theta+\tan\theta|+c_1$

$=\log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a^2}-1}\right|+c_1$

$=\log|x+\sqrt{x^2-a^2}|-\log|a|+c_1$

$=\log|x+\sqrt{x^2-a^2}|+c,$

where $c=c_1-\log|a|$

11. $I=\int \frac{dx}{\sqrt{a^2-x^2}}$

put $x=a\sin\theta$, then $dx=a\cos\theta d\theta$

$\therefore I=\int \frac{a\cos\theta d\theta}{\sqrt{a^2-a^2\sin^2\theta}}=\int \frac{a\cos\theta}{a\cos\theta}d\theta$

$=\int d\theta \;=\theta+c=\sin^{-1}\frac{x}{a}+c$

12. $I=\int \frac{dx}{\sqrt{x^2+a^2}}$

put $x=a\tan\theta$, then $dx=a\sec^2\theta d\theta$

$\therefore I=\int \frac{a\sec^2\theta d\theta}{\sqrt{a^2\tan^2\theta+a^2}}=\int \frac{a\sec^2 d\theta}{a\sec\theta}$

$=\int \sec\theta d\theta \;=\log|\sec\theta+\tan\theta|+c_1$

$=\log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a^2}+1}\right|+c_1$

$=\log|x+\sqrt{x^2+a^2}|-\log|a|+c_1$

$=\log|x+\sqrt{x^2+a^2}|+c$

where $c=c_1-\log|a|$

13. $I=\int \frac{dx}{x^2-6x+13}=\int \frac{dx}{(x-3)^2+4}$

$=\int \frac{dx}{(x-3)^2+(2)^2}$

put $x-3=t$,

then $dx=dt$

$I=\int \frac{dt}{(t)^2+(2)^2}=\frac{1}{2}\tan^{-1}\frac{t}{2}+c$

$=\frac{1}{2}\tan^{-1}\frac{x-3}{2}+c$

14 $$I=\int \frac{x+3}{\sqrt{5-4x-x^2}}dx$$

$$let\ x+3=A.\frac{d}{dx}(5-4x-x^2)+B$$
$$=A(-4-2x)+B$$

equating the cofficients of x and the constant terms from both sides.

$$-2A=1 \quad \text{and}-4A+B=3$$
$$\Rightarrow A=-\frac{1}{2}, \text{and } B=1$$
$$\therefore I=-\frac{1}{2}\int \frac{-4-2x}{\sqrt{5-4x-x^2}}dx+\int \frac{1}{\sqrt{5-4x-x^2}}dx$$
$$=-\frac{1}{2}I_1+I_2 ------ (i)$$

In I_1, put $5-4x-x^2=t, \therefore (-4-2x)dx=dt$

$$\therefore I_1=\int \frac{dt}{\sqrt{t}}=2\sqrt{t}+c_1$$
$$=2\sqrt{5-4x-x^2}+c_1 --- (ii)$$

Now,

$$I_2=\int \frac{dx}{\sqrt{5-4x-x^2}}=\int \frac{dx}{\sqrt{9-(x+2)^2}}$$

put $x+2=t \Rightarrow dx=dt$

$$I_2=\int \frac{dt}{\sqrt{3^2-t^2}} = \sin^{-1}\frac{t}{3}+c_2$$
$$=\sin^{-1}\frac{x+2}{3}+c_2 ------ (iii)$$

Substituting the value of I_1 and I_2 from (ii) and (iii) in (i) we get,

$$I=-\frac{1}{2}(2\sqrt{5-4x-x^2})+\sin^{-1}\frac{x+2}{3}+c_2-\frac{c_1}{2}$$
$$=-\sqrt{5-4x-x^2}+\sin^{-1}\frac{x+2}{3}+c,$$

where $c=-\frac{1}{2}c_1+c_2$

15. $$I=\int \frac{x+2}{\sqrt{x^2-1}}dx$$
$$=\int \left(\frac{x}{\sqrt{x^2-1}}+\frac{2}{\sqrt{x^2-1}}\right)dx$$
$$=\int \frac{x}{\sqrt{x^2-1}}dx+\int \frac{2}{\sqrt{x^2-1}}dx$$
$$=\int \frac{x}{\sqrt{x^2-1}}dx+2\log|x+\sqrt{x^2-1}|+c_1$$

Now,

Let $I_1=\int \frac{x}{\sqrt{x^2-1}}dx = \frac{1}{2}\int \frac{2x}{\sqrt{x^2-1}}dx$

put $x^2-1=t \quad \Rightarrow \quad 2xdx=dt$

$$\therefore I_1=\frac{1}{2}\int \frac{dt}{\sqrt{t}}=\frac{1}{2}\times 2\sqrt{t}+c_2=\sqrt{x^2-1}+c_2$$
$$\therefore I=\sqrt{x^2-1}+2\log|x+\sqrt{x^2-1}|+c_1+c_2$$
$$=\sqrt{x^2-1}+2\log|x+\sqrt{x^2-1}|+c$$

where $c=c_1+c_2$

16. $$I=\int \frac{x}{(x-1)(x-2)(x-3)}dx$$

let $\frac{x}{(x-1)(x-2)(x-3)}$

$$=\frac{A}{(x-1)}+\frac{B}{(x-2)}+\frac{C}{(x-3)} -- (i)$$
$$=\frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$
$$\Rightarrow x=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) -- (ii)$$

put $x=1,2,3$ in (ii) successively we get,

$$1=A(-1)(-2)\Rightarrow A=\frac{1}{2}$$
$$2=B(1)(-1)\Rightarrow -B=2$$
$$3=C(2)(1) \quad \Rightarrow c=\frac{3}{2}$$
$$\therefore I=\int\left[\frac{1}{2(x-1)}-\frac{2}{x-2}+\frac{3}{2(x-3)}\right]dx$$
$$=\frac{1}{2}\int \frac{1}{x-1}dx-2\int \frac{1}{x-2}dx+\frac{3}{2}\int \frac{1}{x-3}dx$$
$$\therefore I=\frac{1}{2}\log|x-1|-2\log|x-2|+\frac{3}{2}\log|x-3|+c$$

17. $I=\int e^x\sin x dx$ ....(1)

Integrating by parts, we get

$$I=e^x\int \sin x\,dx-\int\left\{\frac{d}{dx}(e^x)\int \sin x\,dx\right\}dx$$
$$=e^x(-\cos x)-\int e^x(-\cos x)\,dx$$
$$=-e^x\cos x+\int e^x\cos x\,dx$$
$$=-e^x\cos x+e^x\int \cos x\,dx-\int\left\{\frac{d}{dx}(e^x)\int \cos x\,dx\right\}dx$$
$$=-e^x\cos x+e^x\sin x-\int e^x\sin x\,dx+c_1$$

or, $I=e^x(\sin x-\cos x)-I+c_1$; from (i)

or, $I+I=e^x(\sin x-\cos x)+c_1$

$$2I=e^x(\sin x-\cos x)+c_1$$
$$\therefore I=\frac{e^x}{2}(\sin x-\cos x)+\frac{c_1}{2}$$
$$\therefore \int e^x\sin x dx=\frac{e^x}{2}(\sin x-\cos x)+c;$$

where $c=\frac{c_1}{2}$

18. $$I=\int x\sin^{-1}x\,dx$$

put $x=\sin\theta \Rightarrow dx=\cos\theta d\theta$

$$\therefore I=\int \theta\sin\theta.\cos\theta d\theta = \frac{1}{2}\int \theta\sin 2\theta d\theta$$

Integrating by parts, we get;

$$I=\frac{1}{2}\left[\theta\int \sin 2\theta d\theta-\int\left\{\frac{d(\theta)}{d\theta}\int \sin 2\theta d\theta\right\}d\theta\right]$$
$$=\frac{1}{2}\left[\theta\left(\frac{-\cos 2\theta}{2}\right)-\int \frac{-\cos 2\theta}{2}d\theta\right]$$
$$=\frac{1}{4}\left[-\theta\cos 2\theta+\frac{\sin 2\theta}{2}\right]+c$$
$$=\frac{1}{4}\left[-\theta(1-2\sin^2\theta)+\frac{2\sin\theta\cos\theta}{2}\right]+c$$
$$=\frac{1}{4}[-\sin^{-1}x(1-2x^2)+x\sqrt{1-x^2}]+c$$
$$I=\frac{1}{4}[(2x^2-1)\sin^{-1}x+x\sqrt{1-x^2}]+c$$

19. $I = \int \frac{(x^2+1)e^x}{(x+1)^2}dx$

$= \int \frac{x^2-1+2}{(x+1)^2}e^x dx$

$= \int e^x\left[\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right]dx$

$= \int e^x\left[\frac{x-1}{(x+1)} + \frac{2}{(x+1)^2}\right]dx$

consider $f(x) = \frac{x-1}{x+1}$,

then $f'(x) = \frac{2}{(x+1)^2}$

Thus , the given integrand is of the form $e^x[f(x)+f'(x)]$

Hence, $\int \frac{x^2+1}{(x+1)^2}e^x dx = \frac{x-1}{x+1}.e^x + c$

20. $I = \int \sin^{-1}(\frac{2x}{1+x^2})\,dx$

$= \int 2\tan^{-1}x\,dx$

Integrating by parts , we get

$I = \tan^{-1}x\int 2dx - \int\left[\frac{d}{dx}(\tan^{-1}x)\int 2dx\right]dx$

$= 2x\tan^{-1}x - \int \frac{2x}{1+x^2}dx + c$

$= 2x\tan^{-1}x - \log|1+x^2| + c$

21. $I = \int \sqrt{x^2+2x+5}\,dx$

$= \int \sqrt{(x+1)^2+(2)^2}\,dx$

$= \frac{x+1}{2}\sqrt{(x+1)^2+(2)^2} + \frac{2^2}{2}\log|(x+1)+\sqrt{(x+1)^2+(2)^2}| + c$

$= \frac{x+1}{2}\sqrt{x^2+2x+5} + 2\log|x+1+\sqrt{x^2+2x+5}| + c$

22. $I = \int \sqrt{1+3x-x^2}\,dx$

$= \int \sqrt{1-(x^2-3x)}\,dx$

$= \int \sqrt{1-\left\{(x)^2 - 2.\frac{3}{2}.x + \frac{9}{4} - \frac{9}{4}\right\}}\,dx$

$= \int \sqrt{1+\frac{9}{4}-(x-\frac{3}{2})^2}\,dx$

$= \int \sqrt{\frac{13}{4}-(x-\frac{3}{2})^2}\,dx$

$= \int \sqrt{(\frac{\sqrt{13}}{2})^2-(x-\frac{3}{2})^2}\,dx$

$= (\frac{x-\frac{3}{2}}{2})\sqrt{(\frac{\sqrt{13}}{2})^2-(x-\frac{3}{2})^2} + \frac{(\frac{\sqrt{13}}{2})^2}{2}\sin^{-1}(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}) + c$

$= \frac{2x-3}{4}\sqrt{1+3x-x^2} + \frac{13}{8}\sin^{-1}(\frac{2x-3}{\sqrt{13}}) + c$

23. $I = \int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$

put $\sin^{-1}x = t \quad \Rightarrow \frac{1}{\sqrt{1-x^2}}dx = dt$

$I = \int t\sin t\,dt$

Integrating by parts , we get

$I = t\int \sin t\,dt - \int\left\{\frac{d(t)}{dt}\int \sin t\,dt\right\}dt$

$= t(-\cos t) - \int 1.(-\cos t)\,dt$

$= -t\cos t + \sin t + c$

$= -\sin^{-1}x\sqrt{1-x^2} + x + c$

$\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx = -\sin^{-1}x\sqrt{1-x^2} + x + c$

DEFINITE INTEGRATION (03 MARKS)

1. we have,

$I = \int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}$

$= \int_0^1 \left[\frac{\sqrt{x+1}-\sqrt{x}}{(\sqrt{x+1}+\sqrt{x})(\sqrt{x+1}-\sqrt{x})}\right]dx$

$= \int_0^1 \frac{\sqrt{x+1}-\sqrt{x}}{(x+1-x)}dx$

$= \int_0^1 (\sqrt{x+1}-\sqrt{x})\,dx$

$= \left[\frac{2}{3}(x+1)^{3/2} - \frac{2}{3}x^{3/2}\right]_0^1$

$= \frac{2}{3}[(x+1)^{3/2} - x^{3/2}]_0^1$

$= \frac{2}{3}[(2)^{3/2} - 1^{3/2} - (1^{3/2} - 0)]$

$= \frac{2}{3}(2\sqrt{2}-1-1) = \frac{2}{3}(2\sqrt{2}-2)$

$= \frac{4}{3}(\sqrt{2}-1)$

2. $I=\int_0^{\pi/2}\frac{dx}{1+\sin x} = \int_0^{\pi/2}\frac{dx}{1+\cos(\frac{\pi}{2}-x)}$

$=\int_0^{\pi/2}\frac{dx}{2\cos^2(\frac{\pi}{4}-\frac{x}{2})}$

$=\frac{1}{2}\int_0^{\pi/2}\sec^2(\frac{\pi}{4}-\frac{x}{2})\,dx$

$=-\frac{1}{2}\left[\frac{\tan(\frac{\pi}{4}-\frac{x}{2})}{\frac{1}{2}}\right]_0^{\pi/2}$

$=-\left[\tan(\frac{\pi}{4}-\frac{x}{2})\right]_0^{\pi/2}$

$=-\left[\tan(\frac{\pi}{4}-\frac{\pi}{4})-\tan\frac{\pi}{4}\right]$

$=-[\tan 0-1] = -(0-1)=1$

3. $I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ --- (i)

$=\int_0^{\pi/2}\frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)}+\sqrt{\cos(\frac{\pi}{2}-x)}}dx$

or, $I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ---- (ii)

Adding (i) and (ii) we get,

$2I=\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

$=\int_0^{\pi/2}1dx = [x]_0^{\pi/2}=\frac{\pi}{2}-0$

$=\frac{\pi}{2}$

$\therefore I=\frac{\pi}{4}$ Ans.

4. $I=\int_0^4|x-1|dx$

$=\int_0^1|x-1|dx+\int_1^4|x-1|dx$

$=-\int_0^1(x-1)\,dx+\int_1^4(x-1)\,dx$

$=-\left[\frac{x^2}{2}-x\right]_0^1+\left[\frac{x^2}{2}-x\right]_1^4$

$=-\left[(\frac{1}{2}-1)-0\right]+\left[(\frac{16}{2}-4)-(\frac{1}{2}-1)\right]$

$=\frac{1}{2}+(4+\frac{1}{2})=5$

5. $I=\int_0^{\pi/2}\log\sin x dx$ --- (i)

$=\int_0^{\pi/2}\log\sin(\frac{\pi}{2}-x)\,dx$

$=\int_0^{\pi/2}\log\cos x dx$ --- (ii)

Adding (i) and (ii) we get,

$2I=\int_0^{\pi/2}(\log\sin x+\log\cos x)\,dx$

$=\int_0^{\pi/2}\log(\sin x.\cos x)\,dx$

$=\int_0^{\pi/2}\log(\frac{\sin 2x}{2})\,dx$

$=\int_0^{\pi/2}\log\sin 2x\,dx-\int_0^{\pi/2}\log 2\,dx$

put $2x=t$ in 1st integral

$\therefore\ dx=\frac{1}{2}dt$

when $x=0$ then $t=0$,

when $x=\frac{\pi}{2}$, then $t=\pi$

$\therefore 2I=\frac{1}{2}\int_0^{\pi}\log\sin t\,dt-\log 2[x]_0^{\pi/2}$

$=\frac{2}{2}\int_0^{\pi/2}\log\sin t\,dt-\frac{\pi}{2}\log 2$

$=\int_0^{\pi/2}\log\sin x dx-\frac{\pi}{2}\log 2$ (by changing variable t to x)

$=I-\frac{\pi}{2}\log 2$

$\therefore 2I-I=-\frac{\pi}{2}\log 2$

$\therefore I=-\frac{\pi}{2}\log 2$

6. $I=\int_0^{\pi/4}\log(1+\tan x)\,dx$ --- (i)

$=\int_0^{\pi/4}\log(1+\tan(\frac{\pi}{4}-x))\,dx$

$=\int_0^{\pi/4}\log(1+\frac{1-\tan x}{1+\tan x})\,dx$

$=\int_0^{\pi/4}\log(\frac{1+\tan x+1-\tan x}{1+\tan x})\,dx$

$=\int_0^{\pi/4}\log(\frac{2}{1+\tan x})\,dx$ --- (ii)

Adding (i) and (ii) we get,

$2I=\int_0^{\pi/4}\log(1+\tan x)\,dx$

$+\int_0^{\pi/4}\log(\frac{2}{1+\tan x})\,dx$

$$= \int_0^{\pi/4} \log(1+\tan x)\,dx + \int_0^{\pi/4} [\log 2 - \log(1+\tan x)]dx$$

$$= \int_0^{\pi/4} \log(1+\tan x)\,dx + \int_0^{\pi/4} \log 2\,dx - \int_0^{\pi/4} \log(1+\tan x)\,dx$$

$$= \int_0^{\pi/4} \log 2 dx = \log 2 [x]_0^{\pi/4} = \frac{\pi}{4}\log 2$$

$$\therefore I = \frac{\pi}{8}\log 2$$

7. $I = \int_0^{2\pi} |\cos x|dx$

$$= \int_0^{\pi/2} |\cos x|dx + \int_{\pi/2}^{3\pi/2} |\cos x|dx + \int_{3\pi/2}^{2\pi} |\cos x|dx$$

$$= \int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{3\pi/2} \cos x\,dx + \int_{3\pi/2}^{2\pi} \cos x\,dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{3\pi/2} + [\sin x]_{3\pi/2}^{2\pi}$$

$$= (\sin\frac{\pi}{2} - \sin 0) - (\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}) + (\sin 2\pi - \sin\frac{3\pi}{2})$$

$$= (1-0) - (-1-1) + [0-(-1)]$$

$$= 1+2+1 = 4$$

8. $I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

$$= \int_{\pi/6}^{\pi/3} \left(\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\right)dx \text{ ------ (i)}$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \text{ --- (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_{\pi/6}^{\pi/3} 1 dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

$$\therefore I = \frac{\pi}{12}$$

9. $I = \int_0^{2\pi} \cos^5 x\,dx$

$$= 2\int_0^{\pi} \cos^5 x\,dx \text{ ---- (i)}$$

$[\because \cos(2\pi - x) = \cos x]$

Let $I = \int_0^{\pi} \cos^5 x\,dx$ --- (ii)

$$= \int_0^{\pi} \cos^5(\pi - x)\,dx$$

$$= -\int_0^{\pi} \cos^5 x\,dx \text{ ---- (iii)}$$

$[\because \cos(\pi - x) = -\cos x]$

adding (ii) and (iii), we get,

$2I = 0$

$\therefore I = 0,$

Hence, $\int_0^{2\pi} \cos^5 x\,dx = 2I = 2\times 0 = 0$

10. $I = \int_0^{\pi} \frac{x}{1+\sin x}dx$ ---- (i)

$$= \int_0^{\pi} \frac{\pi - x}{1+\sin(\pi - x)}dx$$

$$= \int_0^{\pi} \frac{\pi - x}{1+\sin x}dx \text{ ---- (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{x+\pi-x}{1+\sin x}dx$$

$$= \pi\int_0^{\pi} \frac{1}{1+\sin x}dx$$

$$= \pi\int_0^{\pi} \frac{1-\sin x}{1-\sin^2 x}dx$$

$$= \pi\int_0^{\pi} \frac{1-\sin x}{\cos^2 x}dx$$

$$= \pi\int_0^{\pi} (\sec^2 x - \sec x.\tan x)\,dx$$

$$= \pi[\tan x - \sec x]_0^{\pi}$$

$$= \pi[(\tan\pi - \sec\pi) - (\tan 0 - \sec 0)]$$

$$= \pi[0-(-1)-(0-1)] = 2\pi$$

$$\therefore I = \pi$$

11. $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$ --- (i)

$$= \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}dx$$

$$= \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\cos x + \sin x}dx \text{ --- (ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{x + \frac{\pi}{2} - x}{\sin x + \cos x}dx$$

$$= \frac{\pi}{2}\int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx$$

$$\therefore I=\frac{\pi}{4}\int_0^{\pi/2}\frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)}dx$$

$$=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\frac{\pi}{4}\right)}dx$$

$$=\frac{\pi}{4\sqrt{2}}\int_0^{\pi/2}\sec\left(x-\frac{\pi}{4}\right)dx$$

$$=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\pi/2}$$

$$=\frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|-\log\left|\sec\frac{\pi}{4}-\tan\frac{\pi}{4}\right|\right]$$

$$=\frac{\pi}{4\sqrt{2}}[\log(\sqrt{2}+1)-\log(\sqrt{2}-1)]$$

$$=\frac{\pi}{4\sqrt{2}}\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)=\frac{\pi}{4\sqrt{2}}\log(\sqrt{2}+1)^2$$

$$\therefore I=\frac{\pi}{2\sqrt{2}}\log(\sqrt{2}+1)$$

12. $$I=\int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx$$

$$=\int_{-a}^{a}\frac{a-x}{\sqrt{a^2-x^2}}dx$$

$$=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx$$

Since, $\frac{x}{\sqrt{a^2-x^2}}$ is an odd function .

Hence $\int_{-a}^{a}\frac{x}{\sqrt{a^2-x^2}}dx=0$

$$\therefore I=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx-0$$

$$=\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx$$

$$=a\int_{-a}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$$

$$=2a\int_0^{a}\frac{dx}{\sqrt{a^2-x^2}}=2a\left[\sin^{-1}\frac{x}{a}\right]_0^a$$

$$=2a[\sin^{-1}1-\sin^{-1}0]=2a\left(\frac{\pi}{2}-0\right)$$

$$I=2a.\frac{\pi}{2}=\pi a$$

DEFINITE INTEGRATION AS THE LIMIT OF SUM

1. We have $\int_a^b f(x)\,dx=\lim_{h\to 0}\sum_{r=1}^{n}hf(a+rh)$

where $nh=b-a$ and $n\to\infty$

Here $f(x)=x^2$, $a=0$, $b=1$

$$\therefore f(a+rh)=f(0+rh)=f(rh)$$
$$=(rh)^2=r^2h^2$$

and $nh=b-a=1-0=1$

$$\therefore \int_0^1 f(x)\,dx=\lim_{h\to 0}h\sum_{r=1}^{n}f(a+rh)$$

$$=\lim_{h\to 0}h\sum_{r=1}^{n}r^2h^2$$

$$=\lim_{h\to 0}h^3\sum_{r=1}^{n}r^2$$

$$=\lim_{h\to 0}\frac{h^3n(n+1)(2n+1)}{6}$$

$$=\lim_{h\to 0}\frac{nh(nh+h)(2nh+h)}{6}$$

$$=\lim_{h\to 0}\frac{1(1+h)(2+h)}{6}$$

$$=\frac{1\times1\times2}{6}=\frac{1}{3}$$

$$\therefore \int_0^1 x^2\,dx=\frac{1}{3}$$

2. From definition

$$\int_a^b f(x)\,dx=\lim_{h\to 0}\sum_{r=1}^{n}hf(a+rh)$$

where $nh=b-a$, $n\to\infty$

Here $f(x)=e^x$, $a=0.$ $b=1$

$\therefore f(a+rh)=f(0+rh)=f(rh)=e^{rh}$

and $nh=b-a=1-0=1$

From (i)

$$\int_0^1 f(x)\,dx=\lim_{h\to 0}h\sum_{r=1}^{n}f(a+rh)$$

$$=\lim_{h\to 0}h\sum_{r=1}^{n}e^{rh}$$

$$=\lim_{h\to 0}h(e^h+e^{2h}+e^{3h}+e^{4h}+..........+e^{nh})$$

$$=\lim_{h\to 0}h\frac{e^h(1-e^{nh})}{1-e^h}$$

$$=\lim_{h\to 0}h\frac{e^h(1-e^1)}{1-e^h}$$

$$=\lim_{h\to 0}\frac{he^h}{-1+e^h}(e-1)$$

$$=\lim_{h\to 0}\frac{e^h(e-1)}{\left(\frac{e^h-1}{h}\right)}$$

$$=\frac{\lim_{h\to 0}e^h(e-1)}{\lim_{h\to 0}\frac{(e^h-1)}{h}}$$

$$=\frac{e^0(e-1)}{1}=(e-1)$$

3. From definition

$$\int_a^b f(x)\,dx == \lim_{h\to 0} \sum_{r=1}^{n} hf(a+rh)$$

Where , $nh = b - a$ and $n \to \infty$

Here $f(x) = x, \quad a = 0, b = 3$

$\therefore f(a+rh) = f(0+rh) = f(rh) = rh$

and $nh = b - a = 3 - 0 = 3$

$$\text{From (i)} \int_0^3 f(x)\,dx = \lim_{h\to 0} h\sum_{r=1}^{n} f(a+rh)$$

$$= \lim_{h\to 0} h\sum_{r=1}^{n} rh$$

$$= \lim_{h\to 0} h^2\sum_{r=1}^{n} r$$

$$= \lim_{h\to 0} h^2 \frac{n(n+1)}{2}$$

$$= \lim_{h\to 0} \frac{nh(nh+h)}{2}$$

$$= \lim_{h\to 0} \frac{3(3+h)}{2}$$

$$= \frac{9}{2} \quad \text{Ans..}$$

4. From definition

$$\int_a^b f(x)\,dx = \lim_{h\to 0} \sum_{r=1}^{n} hf(a+rh) -- (i)$$

Where, $nh = b - a$ and $n \to \infty$

Here, $f(x) = x + e^{2x}, a = 0, b = 4$

$\therefore nh = b - a = 4 - 0 = 4$

$\therefore f(a+rh) = f(0+rh) = f(rh) = rh + e^{2rh}$

$\therefore$ From (i),

$$\int_0^4 (x+e^{2x})dx = \lim_{h\to 0} h\sum_{r=1}^{n}(rh+e^{2rh})$$

$$= \lim_{h\to 0} h^2\sum_{r=1}^{n} r + \lim_{h\to 0} h\sum_{r=1}^{n} e^{2rh}$$

$$= \lim_{h\to 0} h^2 \frac{n(n+1)}{2} + \lim_{h\to 0} h \frac{e^{2h}(e^{2nh}-1)}{e^{2h}-1}$$

$$= \lim_{h\to 0} \frac{nh(nh+h)}{2} + \lim_{h\to 0} \frac{h\,e^{2h}(e^{2\times 4}-1)}{e^{2h}-1}$$

$$= \lim_{h\to 0} \frac{4(4+h)}{2} + \lim_{h\to 0} \frac{e^{2h}(e^8-1)}{\left(\frac{e^{2h}-1}{2h}\right).2}$$

$$= \frac{16}{2} + \frac{e^0(e^8-1)}{1\times 2}$$

$$= 8 + \frac{e^8-1}{2} = \frac{16+e^8-1}{2}$$

$$= \frac{e^8+15}{2} \quad \text{Ans..}$$

Chapter-8	Application of Integrals समाकलनों के अनुप्रयोग

Q 1 **Find the area of the circle $x^2+y^2=a^2$ by integration.**

वृत्त $x^2+y^2=a^2$ का क्षेत्रफल समाकलन द्वारा निकालें।

Q 2 **Find the area of the smaller portion of the circle $x^2+y^2=a^2$ cut off by the line $x=\frac{a}{2}$.**

सरल रेखा $x=\frac{a}{2}$ द्वारा विभाजित वृत्त $x^2+y^2=a^2$ के भागों में से छोटे भाग का क्षेत्रफल निकालें।

Q 3 **Find the area of the region bounded by the parabola $y=x^2$ and $y=|x|$.**

परवलय $y=x^2$ तथा $y=|x|$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात करें।

Q 4 **Find the area bounded by the parabola $y^2=4x$ and the straight line $x+y=3$.**

परवलय $y^2=4x$ तथा सरल रेखा $x+y=3$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात करें।

Q 5 **Find the area of the region bounded by the parabolas $x^2=y$ and $y^2=x$.**

परवलयों $x^2=y$ तथा $y^2=x$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात करें।

Q 6 **Find the area lying above the x-axis and included between the curves $x^2+y^2=8x$ and $y^2=4x$.**

x-अक्ष के ऊपर वृत्त $x^2+y^2=8x$ एवं परवलय $y^2=4x$ के मध्यवर्ती क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Q 7 **Find the area of the region enclosed by the curves $x^2+y^2=1$ and $(x-1)^2+y^2=1$.**

वक्रों $x^2+y^2=1$ तथा $(x-1)^2+y^2=1$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

Q 8 **Using integration, find the area of $\triangle ABC$ whose vertices are A(2,0), B(4,5) and C(6,3).**

समाकलन विधि का प्रयोग करते हुए एक ऐसे त्रिभुज ABC का क्षेत्रफल ज्ञात कीजिए जिसके शीर्ष A(2,0), B(4,5) एवं C(6,3) हैं।

Q 9 **Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$.**

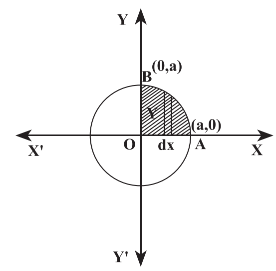
दीर्घ वृत्त $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ एवं रेखा $\frac{x}{a}+\frac{y}{b}=1$ से घिरे लघु क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

5- Marks Solution

1 Ans : -

Whole area enclosed by the given circle

=4(area of the region AOBA bounded by the curve , x-axis and the ordinates x=0 and x=a)



$$=4\int_0^a y\,dx$$

$$=4\int_0^a \sqrt{a^2-x^2}\,dx$$

$$=4\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

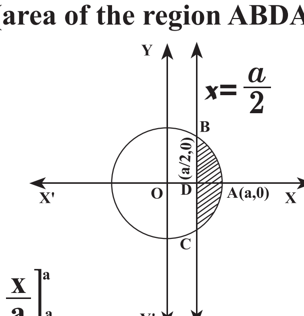
$$=4\left[\left(\frac{a}{2}\times 0+\frac{a^2}{2}\sin^{-1}1\right)-0\right]$$

$$=4\left(\frac{a^2}{2}\right)\left(\frac{\pi}{2}\right)=\pi a^2 \text{ sq. units}$$

2 Ans : -

From the figure ,

Required area = 2(area of the region ABDA)



$$=2\int_{\frac{a}{2}}^{a} y\,dx$$

$$=2\int_{\frac{a}{2}}^{a} \sqrt{a^2-x^2}\,dx$$

$$=2\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{2}}^{a}$$

$$=2\left[\frac{a}{2}\times 0-\frac{a}{4}\times\frac{\sqrt{3a^2}}{2}+\frac{a^2}{2}\left(\sin^{-1}1-\sin^{-1}\frac{1}{2}\right)\right]$$

$$=2\left[\frac{-\sqrt{3}}{8}a^2+\frac{a^2}{2}\left(\frac{\pi}{2}-\frac{\pi}{6}\right)\right]$$

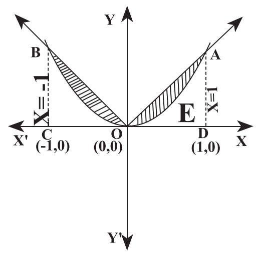
$$=2\left[\frac{-\sqrt{3}}{8}a^2+\frac{a^2}{2}\times\frac{\pi}{3}\right]=a^2\left[\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right] \text{ sq. units}$$

3 Ans : -

Given curves are

$x^2 = y$ ....(1)

and $y = |x|$ ....(2)



From (2)

$$y = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

on solving (1) and (2), we get

curve $x^2 = y$ and $y = x$ intersect each other at O (0,0) and A(1,1).

and curve $x^2 = y$ and $y = -x$ intersect each other at O (0,0) and B(-1,1).

$\therefore$ shaded region is the required region .

Draw AD $\perp$ OX and BC $\perp$ OX'.

$\therefore$ Required area = 2(area of region OEAO)

$$= 2\int_0^1 (y_1 - y_2)dx$$

$$= 2\int_0^1 y\,dx(\text{for the line OA}) - 2\int_0^1 y\,dx(\text{for the parabola})$$

$$= 2\int_0^1 x\,dx - 2\int_0^1 x^2 dx = 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^3}{3}\right]_0^1$$

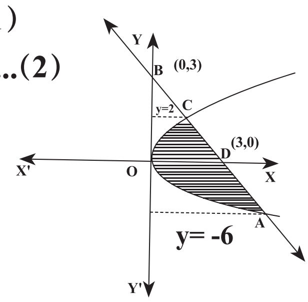
$$= 2\left(\frac{1}{2} - 0\right) - 2\left(\frac{1}{3} - 0\right) = 1 - \frac{2}{3} = \frac{1}{3} \text{ sq. units}$$

4 Ans : -

Given curves are

$y^2 = 4x$ ....(1)

and $x + y = 3$ ....(2)



From (1) and (2)

$y^2 = 4(3 - y)$

$\Rightarrow y^2 + 4y - 12 = 0$

$\Rightarrow y = -6, 2$

$\therefore$ Area of shaded region

i.e. Area of region OCDAO

$$= \int_{-6}^{2} (x_1 - x_2)dy = \int_{-6}^{2}\left[(3 - y) - \frac{y^2}{4}\right]dy$$

$$= \left[3y - \frac{y^2}{2} - \frac{y^3}{12}\right]_{-6}^{2} = \left(6 - 2 - \frac{8}{12}\right) - \left(-18 - 18 - \frac{-216}{12}\right)$$

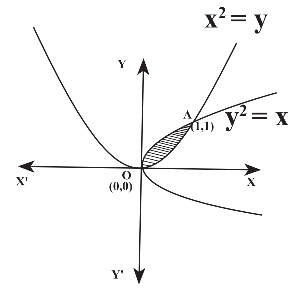
$$= \left(\frac{10}{3} + 18\right) = \frac{64}{3} \text{sq. units}$$

5 Ans ; -

Given parabolas are

$x^2 = y$ ....(1)

and $y^2 = x$ ....(2)



on solving (1) and (2)

points of intersection are O(0,0) and A(1,1).

$\therefore$ Required area

$$= \int_0^1 (y_1 - y_2)dx = \int_0^1 (\sqrt{x} - x^2)dx$$

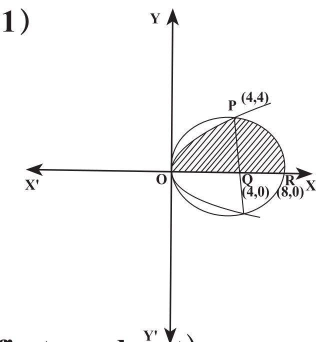
$$= \left[\frac{2}{3}x^{3/2} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3} \text{ sq. units}$$

6 Ans : -

Given curves are

$x^2 + y^2 = 8x$ ....(1)

and $y^2 = 4x$ ....(2)



From (1) and (2)

$x^2 + 4x = 8x$

or , $x^2 - 4x = 0 \Rightarrow x = 0, 4$

$\therefore$ When x = 0 then y = 0

When x = 4 then y = 4(in first quadrant)

$\therefore$ Area of regionOPRQO =

Area of regionOPQO + Area of regionQPRQ

$$= \int_0^4 y\,dx(\text{for parabola}) + \int_4^8 y\,dx(\text{for circle})$$

$$= \int_0^4 2\sqrt{x}\,dx + \int_4^8 \sqrt{8x - x^2}\,dx$$

$$= 2\int_0^4 \sqrt{x}\,dx + \int_4^8 \sqrt{4^2 - (x-4)^2}\,dx$$

$$= 2\left[\frac{2}{3}x^{3/2}\right]_0^4 + \int_0^4 \sqrt{4^2 - t^2}\,dt \qquad \begin{pmatrix} \text{Put } x - 4 = t \\ \therefore dx = dt \\ \text{if } x = 4, t = 0 \\ \text{if } x = 8, t = 4 \end{pmatrix}$$

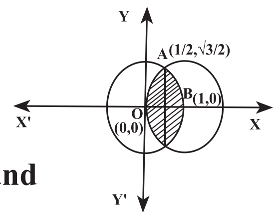
$$= \frac{4}{3}(4^{3/2} - 0) + \left[\frac{t}{2}\sqrt{4^2 - t^2} + \frac{16}{2}\sin^{-1}\frac{t}{4}\right]_0^4$$

$$= \frac{4}{3} \times 8 + [(0 + 8\sin^{-1}1) - (0 + 8\sin^{-1}0)]$$

$$= \frac{32}{3} + \frac{8\pi}{2} - 0 = \frac{32}{3} + 4\pi$$

$$= \frac{4}{3}(8 + 3\pi) \text{ sq. units}$$

7 Ans



Given circles are

$$x^2 + y^2 = 1 \text{ ....(1)}$$

and $(x-1)^2 + y^2 = 1$ **.........(2)**

centre of circle(1) is (0,0) and radius = 1 unit

centre of (2) is (1,0) and radius = 1unit

by (1) − (2), we get $2x - 1 = 0$

$$\Rightarrow x = \frac{1}{2}$$

$\therefore$ **From (1), when** $x = \frac{1}{2}$, $y^2 = \frac{3}{4}$

$$\therefore y = \pm\frac{\sqrt{3}}{2}$$

$\therefore$ **Required area = 2 × area of regionOABO**

$$= 2\int_0^{\sqrt{3}/2} (x_1 - x_2)dy$$

$$= 2\int_0^{\sqrt{3}/2} \left[\sqrt{1-y^2} - \left(1 - \sqrt{1-y^2}\right)\right]dy$$

$$\left[\begin{array}{l}\text{From eqn}(1),\ x = \pm\sqrt{1-y^2}, \therefore \text{ For arc AB } x = \sqrt{1-y^2} \\ \text{From eqn}(2),\ x - 1 = \pm\sqrt{1-y^2}, \therefore \text{ For arc OA } x = 1 - \sqrt{1-y^2}\end{array}\right]$$

$$= 4\int_0^{\sqrt{3}/2} \sqrt{1-y^2}\,dy - 2\int_0^{\sqrt{3}/2} 1dy$$

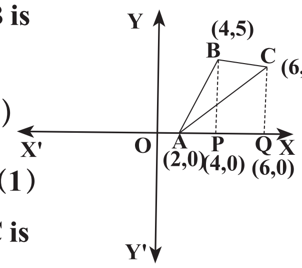
$$= 4\left[\frac{y}{2}\sqrt{1-y^2} + \frac{1}{2}\sin^{-1}y\right]_0^{\sqrt{3}/2} - 2[y]_0^{\sqrt{3}/2}$$

$$= 2\left[\frac{\sqrt{3}}{2}\cdot\frac{1}{2} + \sin^{-1}\frac{\sqrt{3}}{2}\right] - 0 - 2\left(\frac{\sqrt{3}}{2} - 0\right)$$

$$= \frac{\sqrt{3}}{2} + \frac{2\pi}{3} - \sqrt{3}$$

$$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) \text{ sq. units}$$

8 Ans : -



Equation of side AB is

$$y - 0 = \frac{5-0}{4-2}(x-2)$$

or $y = \frac{5}{2}(x-2)$ **........(1)**

Equation of side BC is

$$y - 5 = \frac{3-5}{6-4}(x-4)$$

or , $y - 5 = \frac{-2}{2}(x-4)$

or , $y - 5 = -x + 4$ **or ,** $y = -x + 9$ **........(2)**

Equation of side AC is

$$y - 0 = \frac{3-0}{6-2}(x-2)$$

or , $y = \frac{3}{4}(x-2)$ **............(3)**

From B , draw BP ⊥ OX and from C draw CQ ⊥ OX.

$\therefore$ Area of $\triangle$ ABC = ar($\triangle$APB) + ar(trapezium BPQC) - ar($\triangle$AQC)

$$= \int_2^4 y_{AB}\,dx + \int_4^6 y_{BC}\,dx - \int_2^6 y_{AC}\,dx$$

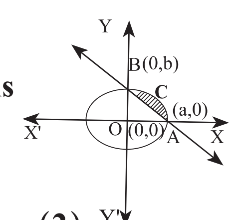
$$= \int_2^4 \frac{5}{2}(x-2)dx + \int_4^6 (-x+9)dx - \int_2^6 \frac{3}{4}(x-2)dx$$

$$= \frac{5}{2}\left[\frac{x^2}{2} - 2x\right]_2^4 + \left[-\frac{x^2}{2} + 9x\right]_4^6 - \frac{3}{4}\left[\frac{x^2}{2} - 2x\right]_2^6$$

$$= \frac{5}{2}[0+2] + [36-28] - \frac{3}{4}[6-(-2)]$$

$$= 5 + 8 - 6 = 7 \text{ sq. units}$$

9.Ans



Given equation of ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ....(1)}$$

and line is $\frac{x}{a} + \frac{y}{b} = 1$ **............(2)**

we have to find the area of shaded region

$\therefore$ **Area of region ABCA** $= \int_0^a (y_1 - y_2)dx$

$$= \int_0^a y dx(\text{for ellipse}) - \int_0^a y dx(\text{for line})$$

$$= \int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx - \int_0^a \frac{b(a-x)}{a}dx$$

$$= \frac{b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a - \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$$

$$= \frac{ab}{2}\left(\sin^{-1}1 - \sin^{-1}0\right) - \left(ab - \frac{ab}{2}\right)$$

$$= \left(\frac{\pi ab}{4} - \frac{ab}{2}\right) \text{ sq. unit}$$

$$\therefore \text{ Required area} = \left(\frac{\pi ab}{4} - \frac{ab}{2}\right)\text{sq.unit}$$

Chapter-9 | Differential Equation
अवकल समीकरण

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 The order of the differential equation
$2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ **is**

अवकल समीकरण $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ की कोटि है:

(a) 2 (b) 1
(c) 0 (d) Not defined (अपरिभाषित)

Q 2 The differential equation representing the family of curve $y^2=2c\left(x+\sqrt{c}\right)$, where c is a positive parameter, is of

$y^2=2c\left(x+\sqrt{c}\right)$, जहाँ C एक घनात्मक प्राचल है,से निरूपित होने वाले वक्र–कुल का अवकल समीकरण निम्नांकित में किस प्रकार का होगा?

(a) order 1,degree 3 (कोटि 1) (घात 3)
(b) order 2,degree 3 (कोटि 2) (घात 3)
(c) order 3,degree 3 (कोटि 3) (घात 3)
(d) order 4,degree 4 (कोटि 4) (घात 4)

Q 3 A solution of the differential equation
$\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$ **is.**

अवकल समीकरण $\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$ का समाधान है:

(a) y = 2 (b) y = 2x
(c) y = 2x-4 (d) $y = 2x^2-4$

Q 4 The order of the differential equation
$\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ **is**

अवकल समीकरण $\frac{d^2y}{dx^2}=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ की कोटि है:

(a) 1 (b) 2
(c) 3 (d) 4

Q 5 The degree of the differential equation
$\left(\frac{d^3y}{dx^3}\right)^2-3\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^4=y^4$ **is**

अवकल समीकरण $\left(\frac{d^3y}{dx^3}\right)^2-3\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^4=y^4$ का घात है :

(a) 4 (b) 2
(c) 3 (d) 1

Q 6 The order and degree of the differential equation $\frac{d^4y}{dx^4}=y+\left(\frac{dy}{dx}\right)^4$ **are respectively**

अवकल समीकरण $\frac{d^4y}{dx^4}=y+\left(\frac{dy}{dx}\right)^4$ की कोटि एवं घात क्रमशः है:

(a) 2,2 (b) 4,1
(c) 2,4 (d) 4,2

Q 7 Integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{x}$ **is**

अवकल समीकरण $\frac{dy}{dx}+y=\frac{1+y}{x}$ का समाकलन गुणांक है:

(a) e^x (b) xe^x
(c) e^x/x (d) x/e^x

Q 8 What is the integrating factor of
$\frac{dy}{dx}+y\sec x=\tan x$

$\frac{dy}{dx}+y\sec x=\tan x$ का समाकलन गुणांक क्या है?

(a) secx+tanx (b) log(secx+tanx)
(c) $e^{\sec x}$ (d) secx

Q 9 The general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$ **is**

अवकल समीकरण $\frac{dy}{dx}=\frac{y}{x}$ का व्यापक हल है:

(a) y=k/x (b) y=kx
(c) y=klogx (d) logy=kx

Q 10 If a and b are the order and degree of the differential equation
$y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=\cos x$ **, then**

यदि a तथा b अवकल समीकरण
$y\frac{dy}{dx}+x^3\left(\frac{d^2y}{dx^2}\right)+xy=\cos x$

के कोटि तथा घात हैं, तो

(a) $a<b$ (b) $a=b$

(c) $a>b$ (d) None of these (इनमें से कोई नहीं)

Very Short Question (2 Marks)

Q 1 Find the differential equation of the family of curves $x^2+y^2=2ax$.

वक्रों के कुल $x^2+y^2=2ax$ का अवकल समीकरण ज्ञात कीजिए।

Q 2 Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

ऐसे परवलयो के कुल का अवकल समीकरण निर्मित कीजिए ,जिनका शीर्ष मूल-बिन्दु पर है और जिनका अक्ष धनात्मक y-अक्ष की दिशा में है।

Solve the following differential equations : -

निम्नलिखित अवकल समीकरणों को हल करें : -

Q 3 $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$

Q 4 $\frac{dy}{dx}=\sqrt{4-y^2}\ ,-2<y<2$

Q 5 $\frac{dy}{dx}=\frac{x-1}{y+2}\ (y\neq -2)$

Q 6 $\frac{dy}{dx}=1-x+y-xy$

Q 7 $\frac{dy}{dx}=(1+x^2)(1+y^2)$

Q 8 $\frac{dy}{dx}=\frac{1-\cos x}{1+\cos x}$

Q 9 $x\frac{dy}{dx}-y=x^2$

Q 10 $x\frac{dy}{dx}=x+y$

Short Question (3 Marks)

Solve ; - (हल करें : -)

Q 1 $\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$

Q 2 $y\log y\,dx-x\,dy=0$

Q 3 $e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$

Q 4 $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$

Q 5 $\frac{dy}{dx}-y=\cos x$

Q 6 $x\frac{dy}{dx}+2y=x^2$

Q 7 $\frac{dy}{dx}+2y=\sin x$

Q 8 $\frac{dy}{dx}+\frac{y}{x}=x^2$

Q 9 $\cos^2 x\frac{dy}{dx}+y=\tan x$

Q 10 $(1+x^2)dy+2xy\,dx=\cot x\,dx\ ,(x\neq 0)$

Q 11 $(x+y)\frac{dy}{dx}=1$

Q 12 $x\frac{dy}{dx}=y-x\tan\frac{y}{x}$

Long Question (5 Marks)

Solve (हल करें) : -

Q 1 $(1+e^{x/y})dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

Q 2. $\left(x\cos\frac{y}{x}+y\sin\frac{y}{x}\right)y\,dx=\left(y\sin\frac{y}{x}-x\cos\frac{y}{x}\right)x\,dy$

Q 3 $y\,dx+x\log\left(\frac{y}{x}\right)dy-2x\,dy=0$

Q 4 $(x^2-1)\frac{dy}{dx}+2xy=\frac{2}{x^2-1}$

Q 5 $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$

Q 6 $\frac{dy}{dx}-2y=\cos 3x$

Q 7 $(x\log x)\frac{dy}{dx}+y=\frac{2}{x}\log x$

Q 8 $\frac{dy}{dx}-3y\cot x=\sin 2x$, given that y=2, when $x=\frac{\pi}{2}$

Q 9 $(1+y^2)dx = (\tan^{-1}y - x)dy$

Q 10 $\frac{dy}{dx} + xy = xy^3$

MCQ 1-Marks Solution

Ans : -

1 - (a) 2 - (a) 3 - (c) 4 - (b) 5 - (b) 6 - (b) 7 - (c)

8 - (a) 9 - (b) 10 - (c)

2-Marks Solution

1 Ans : -

Given curve is $x^2+y^2=2ax$ ....(1)

दिया गया वक्र है $x^2+y^2=2ax$ ....(1)

on differentiating both sides w.r. to x,

दोनों तरफ x के सापेक्ष अवकलन करने पर,

$$2x + 2y\frac{dy}{dx} = 2a \quad ....(2)$$

$\therefore$ From (1) and (2)

(1) तथा (2) से ,

$$x^2 + y^2 = \mathbf{x}\left(2x + 2y\frac{dy}{dx}\right)$$

or , $x^2 + y^2 = 2x^2 + 2xy\frac{dy}{dx}$

or , $2xy\frac{dy}{dx} + x^2 - y^2 = 0$

which is the required differential equation .

यही अभीष्ट अवकल समीकरण है।

2 Ans : -

Let P denote the family of parabolas and let (0,a) be the focus of a member of the given family , where a is an arbitrary constant.

माना कि P परवलय के कुल है तथा (0,a) नाभि का नियामक है,जहाँ a कोई स्वेच्छ अचर है।

$\therefore$ Equation of family P is

अतः वक्रों के कुल P का समीकरण है

$$x^2=4ay \quad ....(1)$$

Differentiating both sides of equation (1) w.r. to x, we get,

समीकरण (1) को दोनों तरफ x के सापेक्ष अवकलित करने पर,

$$2x = 4a\frac{dy}{dx}$$

$\therefore 2x = \frac{x^2}{y}\frac{dy}{dx}$ [from (1), $4a = \frac{x^2}{y}$]

or , $2y = x\frac{dy}{dx}$

or , $x\frac{dy}{dx} - 2y = 0$

which is the differential equation of the given family of parabolas.

यही दिये गए परवलयों के कुल का अवकल समीकरण है।

3 Ans : -

Given differential equation is $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

दिया गया अवकल समीकरण है

or , $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

Integrating both sides, we get

दोनों तरफ समाकलित करने पर,

$$\int\frac{dy}{1+y^2} = \int\frac{dx}{1+x^2}$$

or, $\tan^{-1}y = \tan^{-1}x + c$

or, $\tan^{-1}y - \tan^{-1}x = c$

or , $\tan^{-1}\frac{y-x}{1+yx} = c$

or , $\frac{y-x}{1+xy} = \tan c$

or , $\frac{y-x}{1+xy} = k$ (where $k = \tan c$)

which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $\frac{dy}{dx} = \sqrt{4-y^2}$

or , $\frac{dy}{\sqrt{4-y^2}} = dx$

on integrating both sides, we get

दोनों तरफ समाकलित करने पर

$$\int\frac{dy}{\sqrt{4-y^2}} = \int dx$$

or , $\int\frac{dy}{\sqrt{2^2-y^2}} = \int dx$

or , $\sin^{-1}\left(\frac{y}{2}\right) = x + c$

$$\text{or},\ \frac{y}{2} = \sin(x + c)$$

$$\therefore y = 2\sin(x + c)$$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $\frac{dy}{dx} = \frac{x - 1}{y + 2}$

or $(y + 2)dy = (x - 1)dx$

दोनो तरफ समाकलित करने पर

integrating both sides,we get

$$\int (y + 2)dy = \int (x - 1)dx$$

$$\text{or},\ \frac{y^2}{2} + 2y = \frac{x^2}{2} - x + c$$

$$\text{or},\ \frac{y^2 + 4y}{2} = \frac{x^2 - 2x + 2c}{2}$$

$y^2 + 4y = x^2 - 2x + k$, where $k = 2c$

$y^2 + 4y - x^2 + 2x = k$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} = 1 - x + y - xy$$

$$\text{or},\ \frac{dy}{dx} = 1(1 - x) + y(1 - x)$$

$$\text{or},\ \frac{dy}{dx} = (1 - x)(1 + y)$$

$$\text{or},\ \frac{dy}{1 + y} = (1 - x)dx$$

दोनो तरफ समाकलित करने पर

integrating both sides , we get

$$\int \frac{dy}{1 + y} = \int (1 - x)dx$$

$$\text{or},\ \log|1 + y| = x - \frac{x^2}{2} + c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

7 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

$$\text{or},\ \frac{dy}{1 + y^2} = (1 + x^2)dx$$

दोनो तरफ समाकलित करने पर

integrating both sides , we get

$$\int \frac{dy}{1 + y^2} = \int (1 + x^2)dx$$

$$\text{or},\ \tan^{-1} y = x + \frac{x^3}{3} + c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans : -

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$\text{or},\ dy = \left(\frac{1 - \cos x}{1 + \cos x}\right)dx$$

$$\text{or},\ dy = \frac{2\sin^2 x/2}{2\cos^2 x/2}dx$$

$$\text{or},\ dy = \tan^2 x/2\, dx$$

$$\text{or},\ dy = \left(\sec^2 x/2 - 1\right)dx$$

दोनो तरफ समाकलित करने पर

integrating both sides , we get

$$\int dy = \int \left(\sec^2 x/2 - 1\right)dx$$

$$\text{or},\ y = \frac{\tan x/2}{\frac{1}{2}} - x + c$$

$$\text{or},\ y = 2\tan x/2 - x + c$$

Which is the required genreal solution

यही अभीष्ट व्यापक हल है।

9 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $x\frac{dy}{dx} - y = x^2$

or $\frac{dy}{dx} - \frac{1}{x}.y = x$ ....(1)

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P=-\frac{1}{x}$, तथा $Q=X$

which is a linear differential equation of the from $\frac{dy}{dx}+Py=Q$, where $P=-\frac{1}{x}$, $Q=x$

Now I.F.$=e^{\int Pdx}=e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log\frac{1}{x}}$

$\Rightarrow$ I.F $=\frac{1}{x}$

$\therefore$ Solution of the d.e.(1) will be

अतः अवकल समीकरण (1) का हल होगा : -

$$y\cdot\frac{1}{x}=\int\frac{1}{x}\cdot x dx+c$$

or, $\frac{y}{x}=x+c \quad \Rightarrow y=x^2+cx$

यही अभीष्ट व्यापक हल है।

Which is the required general solution .

10 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $x\frac{dy}{dx}=x+y$

or, $\frac{dy}{dx}=1+\frac{y}{x}$

or, $\frac{dy}{dx}-\frac{1}{x}.y=1$ ....(1)

which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=-\frac{1}{x}$, $Q=1$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है जहाँ $P=-\frac{1}{x}$, $Q=1$

Now, I.F.$=e^{\int Pdx}=e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log\frac{1}{x}}=\frac{1}{x}$

$\therefore$ Solution of the d.e.(1) will be

अतः अवकल समीकरण (1) का हल होगा : -

$$y.\frac{1}{x}=\int\frac{1}{x}.1dx+c$$

or, $\frac{y}{x}=\log|x|+c$

or, $y=x\log|x|+cx$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

3-Marks Solution

1 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$\sec^2x\tan y dx+\sec^2y\tan x dy=0$

dividing by tanx.tany , we get

$$\frac{\sec^2x}{\tan x}dx+\frac{\sec^2y}{\tan y}dy=0$$

समाकलित करने पर

Integrating , we get

$$\int\frac{\sec^2x}{\tan x}dx+\int\frac{\sec^2y}{\tan y}dy=\log c$$

or, $\log|\tan x|+\log|\tan y|=\log c$

or, $\log|\tan x.\tan y|=\log c$

or, $|\tan x.\tan y|=c$

$\therefore \tan x.\tan y=\pm c$

or, $\tan x.\tan y=k$ where $k=\pm c$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

2 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is $y\log y dx-xdy=0$

or, $xdy=y\log y dx$

or, $\frac{dy}{y\log y}=\frac{dx}{x}$ ....(1)

दोनों तरफ समाकलित करने पर

Integrating both sides , we get

$$\int\frac{dy}{y\log y}=\int\frac{dx}{x}$$

or, $\log(\log y)=\log|x|+\log|c|$

$\Rightarrow \log(\log y)=\log|cx|$

$\Rightarrow \log y=cx \quad \Rightarrow y=e^{cx}$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

3 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$e^x\tan y dx+(1-e^x)\sec^2y dy=0$

or, $\frac{e^x}{1-e^x}dx+\frac{\sec^2y}{\tan y}dy=0$

समाकलित करने पर

Integrating , we get

$$\int \frac{e^x}{1-e^x}dx + \int \frac{\sec^2 y}{\tany}dy = \log c$$

or , $-\log(1-e^x) + \log \tan y = \log c$

or , $\log\frac{\tan y}{1-e^x} = \log c$

or , $\frac{\tan y}{1-e^x} = c \quad \Rightarrow \tan y = c(1-e^x)$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans ; -

दिया गया अवकल समीकरण है

Given differential equation is

$$x dy - y dx = \sqrt{x^2+y^2}\,dx$$

or , $x dy = \left(y + \sqrt{x^2+y^2}\right)dx$

or , $\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$ ....(1)

Which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

put $y = vx \quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ Equation (1) becomes

$\therefore$ समीकरण(1) हो जाता है

$$v + x\frac{dv}{dx} = \frac{vx+\sqrt{x^2+v^2x^2}}{x}$$

or , $x\frac{dv}{dx} = \sqrt{1+v^2}$ or , $\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$

समाकलित करने पर

Integrating , we get

$$\log\left|v+\sqrt{1+v^2}\right| = \log|x| + \log k$$

$$\Rightarrow \left|v+\sqrt{1+v^2}\right| = k|x|$$

$$\Rightarrow v+\sqrt{1+v^2} = \pm kx = cx \text{ , where } c = \pm k$$

$$\therefore \frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}} = cx$$

$$\Rightarrow y+\sqrt{x^2+y^2} = cx^2$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans ; -

दिया गया अवकल समीकरण है

Given differential equation is $\frac{dy}{dx} - y = \cos x$ ...(1)

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = -1, \quad Q = \cos x$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है जहाँ $P = -1, Q = \cos x$

$\therefore$ I.F $= e^{\int p dx} = e^{\int -1 dx} = e^{-x}$

$\therefore$ Solution of d.e. (1) , will be

अतः अवकल समीकरण (1) का हल होगा : -

$$y.e^{-x} = \int e^{-x}.\cos x dx + c \text{ ....(2)}$$

Let $I = \int e^{-x}\cos x dx$

$$= \cos x.(-e^{-x}) - \int -e^{-x}(-\sin x)dx$$

$$= -e^{-x}\cos x - \int e^{-x}\sin x dx$$

$$= -e^{-x}\cos x - \left[\sin x(-e^{-x}) - \int \cos x(-e^{-x})dx\right]$$

$$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x dx$$

or , $I = e^{-x}(\sin x - \cos x) - I$

or , $2I = e^{-x}(\sin x - \cos x)$

$\therefore I = \frac{e^{-x}(\sin x - \cos x)}{2}$

$\therefore$ From eqn (2) , $y.e^{-x} = \frac{e^{-x}(\sin x - \cos x)}{2} + c$

$\therefore \ y = \frac{\sin x - \cos x}{2} + ce^x$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans : -Given differential equation is

दिया गया अवकल समीकरण है

$$x\frac{dy}{dx} + 2y = x^2 \quad \text{or , } \frac{dy}{dx} + \frac{2}{x}y = x \text{ ......(1)}$$

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{2}{x}$, $Q = x$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P = \frac{2}{x}$, $Q = x$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

$\therefore$ Solution of d.e. (1) will be

$\therefore$ अवकल समीकरण (1) का हल होगा

$$y.x^2 = \int x^2.x dx + c$$

or , $y.x^2 = \frac{x^4}{4} + c$

$\therefore\ y = \frac{x^2}{4} + \frac{c}{x^2}$

Which is the general solution of given d.e.

यही अभीष्ट व्यापक हल है।

7 Ans : -

दिया गया अवकल समीकरण है

Given d.e. is $\frac{dy}{dx} + 2y = \sin x$ ....(1)

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = 2$, $Q = \sin x$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P = 2$, $Q = \sin x$

$\therefore$ I.F. $= e^{\int Pdx} = e^{\int 2dx} = e^{2x}$

$\therefore$ Solution of d.e. (1) will be

$\therefore$ अवकल समीकरण (1) का हल होगा

$y.e^{2x} = \int e^{2x}.\sin x dx + c$ ....(2)

Let $I = \int e^{2x} \sin x dx$

$= \sin x.\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} \cdot \cos x dx$

$= \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2}\left[\cos x\left(\frac{e^{2x}}{2}\right) - \int \frac{e^{2x}}{2} \cdot (-\sin x) dx\right]$

$= \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cdot \cos x}{4} - \frac{1}{4}\int e^{2x} \sin x$

or , $I = \frac{e^{2x}(2\sin x - \cos x)}{4} - \frac{1}{4}I$

or , $I + \frac{1}{4}I = \frac{e^{2x}(2\sin x - \cos x)}{4}$

or , $\frac{5}{4}I = \frac{e^{2x}(2\sin x - \cos x)}{4}$

$\therefore I = \frac{e^{2x}(2\sin x - \cos x)}{5}$

$\therefore$ From eqn(2)

$\therefore$ समी.(2)से,

$y \cdot e^{2x} = \frac{e^{2x}(2\sin x - \cos x)}{5} + c$

$\therefore\ y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$

which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans : -

दिया गया अवकल समीकरण है

Given d.e. is $\frac{dy}{dx} + \frac{y}{x} = x^2$ ....(1)

which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x}$, $Q = x^2$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है जहाँ $P = \frac{1}{x}$, $Q = x^2$

$\therefore$ I.F. $= e^{\int Pdx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$y \cdot x = \int x \cdot x^2 dx + c$

or , $xy = \int x^3 dx + c$

or , $xy = \frac{x^4}{4} + c$

or , $y = \frac{x^3}{4} + \frac{c}{x}$

which is the required general solution .

यही अभीष्ट व्यापक हल है।

9 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$\cos^2 x \frac{dy}{dx} + y = \tan x$

or , $\frac{dy}{dx} + \sec^2 x.y = \sec^2 x.\tan x$ ....(1)

which is a linear d.e. of the form $\frac{dy}{dx} + Py = Q$

where $P = \sec^2 x$, $Q = \sec^2 x.\tan x$

यह $\frac{dy}{dx} + Py = Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P = \sec^2 x$, $Q = \sec^2 x \tan x$

Now , I.F. $= e^{\int Pdx} = e^{\int \sec^2 x dx} = e^{\tan x}$

$\therefore$ Solution of eqn(1) will be

अतः समीकरण (1) का हल होगा : -

$y \cdot e^{\tan x} = \int e^{\tan x} \cdot \sec^2 x \cdot \tan x dx + c$

put $\tan x = t$ $\quad \therefore \sec^2 x dx = dt$

$\therefore\ y.e^{\tan x} = \int t \cdot e^t dt + c$

$= t\int e^t dt - \int\left(\frac{d(t)}{dt}\int e^t dt\right)dt + c$

$= te^t - \int e^t dt + c$

$= te^t - e^t + c$

$= tanx.e^{tanx} - e^{tanx} + c$

$= e^{tanx}(tanx - 1) + c$

$\therefore y = (tanx - 1) + ce^{-tanx}$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

10 Ans : -

दिया गया अवकल समीकरण है:–

Given differential equation is

$(1 + x^2)dy + 2xydx = cotxdx$

or , $(1 + x^2)\frac{dy}{dx} + 2xy = cotx$

or , $\frac{dy}{dx} + \frac{2x}{1 + x^2} \cdot y = \frac{cotx}{1 + x^2}$ ....(1)

which is a linear d.e. of the form $\frac{dy}{dx} + Py = Q$

where $P = \frac{2x}{1 + x^2}$, $Q = \frac{cotx}{1 + x^2}$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है जहाँ** $P = \frac{2x}{1 + x^2}$, $Q = \frac{cotx}{1 + x^2}$

Now , I.F. $= e^{\int Pdx} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1 + x^2$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$y(1 + x^2) = \int (1 + x^2) \cdot \frac{cotx}{1 + x^2} dx + c$

or , $y(1 + x^2) = \int cotxdx + c$

or , $y(1 + x^2) = \log|sinx| + c$

$\therefore \quad y = (1 + x^2)^{-1}\log|sinx| + c(1 + x^2)^{-1}$

Which is required general solution.

यही अभीष्ट व्याप्क हल है।

11 Ans : -

$(x + y)\frac{dy}{dx} = 1$, or , $\frac{dx}{dy} = x + y$

or , $\frac{dx}{dy} - x = y$ ....(1)

which is a linear d.e. of the form

$\frac{dx}{dy} + Px = Q$, where , $P = -1$, $Q = y$

यह $\frac{dx}{dy} + Px = Q$ **के रूप का रैखिक अवकल समीकरण है जहाँ** $P = -1$, $Q = y$

$\therefore$ I.F. $= e^{\int Pdy} = e^{\int -1dy} = e^{-y}$

$\therefore$ Solution of eqn(1) will be

अतः समीकरण (1) का हल होगा : -

$x \cdot e^{-y} = \int ye^{-y}dy + c$

or , $x \cdot e^{-y} = -ye^{-y} - \int -e^{-y}dy + c$

$= -ye^{-y} - e^{-y} + c \quad = -e^{-y}(1 + y) + c$

$\therefore \; x = -(1 + y) + ce^{y}$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

12 Ans : - Given d.e. is $x\frac{dy}{dx} = y - xtan\frac{y}{x}$

दिया गया अवकल समीकरण है

$\frac{dy}{dx} = \frac{y}{x} - tan\frac{y}{x}$ ....(1)

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

$\therefore$ Put y=vx

$\therefore \; \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ Equation (1) becomes

समीकरण (1) हो जाता है

$v + x\frac{dv}{dx} = v - tanv$

$\Rightarrow x\frac{dv}{dx} = -tanv$

or , $\frac{dv}{tanv} = -\frac{dx}{x}$

$\Rightarrow \int \frac{dv}{tanv} = -\int \frac{dx}{x}$

or , $\int cotvdv = -\log|x| + logc$

or , $\log(sinv) + \log|x| = logc$

$\Rightarrow \log|xsinv| = logc$

$\Rightarrow |xsinv| = c$

$\Rightarrow xsinv = \pm c = k$ (say)

$\Rightarrow xsin\frac{y}{x} = k$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5-Marks Solution

1 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$$

or , $$\frac{dx}{dy} = -\frac{e^{x/y}\left(1-\frac{x}{y}\right)}{1+e^{x/y}} \quad ....(1)$$

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

Put $x = vy$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore$ From eqn(1) ,

अतः समीकरण (1) से : -

$$v + y\frac{dv}{dy} = -\frac{e^v(1-v)}{1+e^v}$$

$$\therefore\ y\frac{dv}{dy} = -\frac{e^v(1-v)}{1+e^v} - v$$

$$= \frac{-e^v(1-v) - v(1+e^v)}{1+e^v}$$

or , $$y\frac{dv}{dy} = -\frac{v+e^v}{1+e^v}$$

or , $$\frac{1+e^v}{v+e^v}dv = -\frac{dy}{y}$$

$$\Rightarrow \int \frac{1+e^v}{v+e^v}dv = -\int \frac{dy}{y}$$

$$\Rightarrow \log|v+e^v| = -\log|y| + c$$

$$\Rightarrow \log|y(v+e^v)| = c$$

$$\Rightarrow |y(v+e^v)| = e^c$$

$$\Rightarrow y\left(\frac{x}{y} + e^{x/y}\right) = \pm e^c = k(say)$$

$$\therefore \quad x + ye^{x/y} = k$$

Which is the required general solution .

यही अभीष्ट व्यापक हल है।

2 Ans : -

दिया गया अवकल समीकरण है–

Given differential equation is

$$y\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right)dx = x\left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)dy$$

or , $$y\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right) = x\left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)\frac{dy}{dx}$$

or , $$\frac{dy}{dx} = \frac{\left\{x\cos\frac{y}{x} + y\sin\left(\frac{y}{x}\right)\right\}y}{\left\{y\sin\frac{y}{x} - x\cos\frac{y}{x}\right\}x}$$

dividing num. and den. by x^2 , we get

अंश तथा हर को x^2 से विभाजित करने पर,

$$\frac{dy}{dx} = \frac{\left\{\cos\frac{y}{x} + \frac{y}{x}\sin\left(\frac{y}{x}\right)\right\}\frac{y}{x}}{\left\{\frac{y}{x}\sin\frac{y}{x} - \cos\frac{y}{x}\right\}} \quad ....(1)$$

Put $y = vx$, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore$ From (1) , $$v + x\frac{dv}{dx} = \frac{(\cos v + v\sin v)v}{v\sin v - \cos v}$$

or , $$x\frac{dv}{dx} = \frac{v(\cos v + v\sin v)}{v\sin v - \cos v} - v$$

or , $$x\frac{dv}{dx} = \frac{v(\cos v + v\sin v - v\sin v + \cos v)}{v\sin v - \cos v}$$

$$= \frac{2v\cos v}{v\sin v - \cos v}$$

or , $$\frac{v\sin v - \cos v}{v\cos v}dv = \frac{2dx}{x}$$

or , $$\int \frac{v\sin v - \cos v}{v\cos v}dv = 2\int \frac{1}{x}dx$$

$$\Rightarrow \int\left(\tan v - \frac{1}{v}\right)dv = 2\log|x| + \log c$$

or , $-\log|\cos v| - \log|v| = 2\log|x| + \log c$

or , $-\log|v\cos v| = 2\log|x| + \log c$

or , $2\log|x| + \log|v\cos v| = -\log c$

or , $\log|x^2 v\cos v| = \log k \quad (-\log c = \log k)$

$\Rightarrow |x^2 v\cos v| = k \Rightarrow x^2 v\cos v = \pm k = c$

(Where $\pm k = c$)

$$\Rightarrow x^2\frac{y}{x}\cos\frac{y}{x} = c \quad \Rightarrow xy\cos\frac{y}{x} = c$$

Which is the required general solution.
यही अभीष्ट व्यापक हल है।

3 Ans : -दिया गया अवकल समीकरण है

Given differential equation is

$$ydx + x\log\left(\frac{y}{x}\right)dy - 2xdy = 0 \quad ....(1)$$

or , $$y\frac{dx}{dy} = 2x - x\log\left(\frac{y}{x}\right)$$

or , $$\frac{dx}{dy} = 2\frac{x}{y} - \frac{x}{y}\log\left(\frac{y}{x}\right)$$

or , $$\frac{dx}{dy} = 2\cdot\frac{x}{y} + \frac{x}{y}\log\left(\frac{x}{y}\right) \quad ....(2)$$

$$\left(\because \ \log a = -\log\frac{1}{a}\right)$$

which is a homogeneous differential equation

यह एक समघातीय अवकल समीकरण है।

Put $x = vy$, then $\frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore$ From eqn(2) , $v + y\frac{dv}{dy} = 2v + v\log v$

$$\Rightarrow y\frac{dv}{dy} = v + v\log v$$

$$\Rightarrow \frac{dv}{v(1+\log v)} = \frac{dy}{y}$$

$$\Rightarrow \int \frac{dv}{v(1+\log v)} = \int \frac{dy}{y}$$

$$\Rightarrow \log|1+\log v| = \log|y| + c$$

$$\Rightarrow \log\left|\frac{1+\log v}{y}\right| = c$$

$$\Rightarrow \left|\frac{1+\log v}{y}\right| = e^c$$

$$\therefore \quad \frac{1+\log v}{y} = \pm e^c = k(\text{say})$$

$$\Rightarrow 1 + \log\frac{x}{y} = ky ,$$

Where k is an arbitrary constant.

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

4 Ans : -

दिया गया अवकल समीकरण है

Given d.e. is $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$

or , $\frac{dy}{dx} + \frac{2x}{x^2-1}\cdot y = \frac{2}{(x^2-1)^2}$ ....(1)

which is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

where $P = \frac{2x}{x^2-1}$, $Q = \frac{2}{(x^2-1)^2}$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = \frac{2x}{x^2-1}$, $Q = \frac{2}{(x^2-1)^2}$

$\therefore$ I.F. $= e^{\int Pdx} = e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2-1$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$$y(x^2-1) = \int (x^2-1)\cdot\frac{2}{(x^2-1)^2}\cdot dx + c$$

$$= \int \frac{2}{x^2-1}dx + c$$

$$\therefore \ y(x^2-1) = 2\cdot\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + c$$

or , $y(x^2-1) = \log\left|\frac{x-1}{x+1}\right| + c$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

5 Ans ; -

दिया गया अवकल समीकरण है

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + y = \tan^{-1}x$$

or , $\frac{dy}{dx} + \frac{1}{1+x^2}\cdot y = \frac{\tan^{-1}x}{1+x^2}$ ....(1)

which is a linear d.e. of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{1+x^2}$, $Q = \frac{\tan^{-1}x}{1+x^2}$

यह $\frac{dy}{dx} + Py = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = \frac{1}{1+x^2}$, $Q = \frac{\tan^{-1}x}{1+x^2}$

Now , I.F. $= e^{\int Pdx} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा : -

$$y.e^{\tan^{-1}x} = \int e^{\tan^{-1}x}\cdot\frac{\tan^{-1}x}{1+x^2}dx + c$$

Put $\tan^{-1}x = t$ $\quad \therefore \frac{1}{1+x^2}dx = dt$

$$\therefore \ y.e^{\tan^{-1}x} = \int te^t dt + c$$

$$= t\int e^t dt - \int\left\{\frac{d(t)}{dt}\int e^t dt\right\}dt + c$$

$$= te^t - \int e^t dt + c$$

$$= te^t - e^t + c$$

$$\therefore \ y\cdot e^{\tan^{-1}x} = (t-1)e^t + c$$

or , $y\cdot e^{\tan^{-1}x} = (\tan^{-1}x - 1)e^{\tan^{-1}x} + c$

$$\therefore \ y = \tan^{-1}x - 1 + ce^{-\tan^{-1}x}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

6 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$\frac{dy}{dx} - 2y = \cos 3x$ ....(1)

which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=-2$, $Q=\cos3x$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P=-2$, $Q=\cos3x$

Now, I.F. $=e^{\int Pdx}=e^{\int -2dx}=e^{-2x}$

$\therefore$ Solution of the equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y\cdot e^{-2x}=\int e^{-2x}\cos3x\,dx+c \quad ....(2)$$

Let $I=\int e^{-2x}\cos3x\,dx$

then

$$I=\cos3x\int e^{-2x}dx-\int\left\{\frac{d}{dx}(\cos3x)\int e^{-2x}dx\right\}dx$$

$$=\frac{-e^{-2x}\cos3x}{2}-\int(-3\sin3x)\left(\frac{-e^{-2x}}{2}\right)dx$$

$$=\frac{-e^{-2x}\cos3x}{2}-\frac{3}{2}\int e^{-2x}\sin3x\,dx$$

$$=\frac{-e^{-2x}\cos3x}{2}-\frac{3}{2}\left[-\sin3x\cdot\frac{e^{-2x}}{2}-\int 3\cos3x\cdot\frac{e^{-2x}}{-2}dx\right]$$

$$=\frac{-e^{-2x}\cos3x}{2}+\frac{3e^{-2x}\sin3x}{4}-\frac{9}{4}\int e^{-2x}\cos3x\,dx$$

or, $I=\frac{-2e^{-2x}\cos3x+3e^{-2x}\sin3x}{4}-\frac{9}{4}I$

or, $I+\frac{9}{4}I=\frac{e^{-2x}(-2\cos3x+3\sin3x)}{4}$

or, $\frac{13}{4}I=\frac{e^{-2x}(-2\cos3x+3\sin3x)}{4}$

$\Rightarrow I=\frac{1}{13}e^{-2x}(-2\cos3x+3\sin3x)$

$\therefore$ From eqn(2),

$$y\cdot e^{-2x}=\frac{e^{-2x}}{13}(3\sin3x-2\cos3x)+c$$

$$\Rightarrow y=\frac{1}{13}(3\sin3x-2\cos3x)+ce^{2x}$$

Which is the required general solution.
यही अभीष्ट व्यापक हल है।

7 Ans :-

दिया गया अवकल समीकरण है

Given d.e. is $x\log x\cdot\frac{dy}{dx}+y=\frac{2}{x}\log x$

or, $\frac{dy}{dx}+\frac{1}{x\log x}\cdot y=\frac{2}{x^2}$ ....(1)

which is a linear d.e.of the form $\frac{dy}{dx}+Py=Q$

where $P=\frac{1}{x\log x}$ and $Q=\frac{2}{x^2}$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P=\frac{1}{x\log x}$ and $Q=\frac{2}{x^2}$

Now, I.F. $=e^{\int Pdx}=e^{\int\frac{1}{x\log x}dx}=e^{\log(\log x)}=\log x$

$\therefore$ Solution of equation (1) will be

अतः समीकरण (1) का हल होगा :-

$$y\cdot\log x=\int\frac{2}{x^2}\cdot\log x\,dx+c$$

$$=2\left[\log x\int\frac{1}{x^2}dx-\int\left\{\frac{d(\log x)}{dx}\int\frac{1}{x^2}dx\right\}dx\right]+C$$

$$=2\left[\log x\left(-\frac{1}{x}\right)-\int\frac{1}{x}\left(-\frac{1}{x}\right)dx\right]+c$$

$$=-\frac{2\log x}{x}+2\int\frac{1}{x^2}dx+c$$

$$=-\frac{2\log x}{x}-\frac{2}{x}+c$$

$$=-\frac{2}{x}(\log x+1)+c$$

$$\therefore\ y\cdot\log x=-\frac{2}{x}(\log x+1)+c$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

8 Ans :-

दिया गया अवकल समीकरण है

Given differential equation is

$\frac{dy}{dx}-3\cot x.y=\sin2x$ ....(1)

which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=-3\cot x$, $Q=\sin2x$

यह $\frac{dy}{dx}+Py=Q$ के रूप का रैखिक अवकल समीकरण है, जहाँ $P=-3\cot x$ and $Q=\sin2x$

Now, I.F. $=e^{\int Pdx}=e^{\int -3\cot x dx}=e^{-3\log\sin x}=e^{\log(\sin x)^{-3}}$

$$=(\sin x)^{-3}=\frac{1}{\sin^3x}$$

$\therefore$ Solution of the given differential equation(1) will be

अतः दिए गए अवकल समीकरण (1) का हल होगा :-

$$y\times\frac{1}{\sin^3x}=\int\left(\sin2x\times\frac{1}{\sin^3x}\right)dx+c$$

or, $\frac{y}{\sin^3x}=\int\frac{2\sin x\cdot\cos x}{\sin^3x}dx+c$

$$=2\int\frac{\cos x}{\sin^2x}dx+c$$

$$= 2\int \frac{dt}{t^2} + c \quad \text{where } \sin x = t$$
$$= -\frac{2}{t} + c = -\frac{2}{\sin x} + c$$
$$\therefore \frac{y}{\sin^3 x} = -\frac{2}{\sin x} + c$$
$$\therefore y = -2\sin^2 x + c\sin^3 x \quad ....(2)$$

Now , given $y = 2$ when $x = \frac{\pi}{2}$

$\therefore$ From eqn(2) , $2 = -2 + c \Rightarrow c = 4$

$$\therefore \quad y = -2\sin^2 x + 4\sin^3 x$$

Which is the required particular solution.

यही अभीष्ट विशेष हल है।

9 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$(1 + y^2)dx = (\tan^{-1} y - x)dy$$
$$\text{or , } \frac{dx}{dy} = \frac{\tan^{-1} y - x}{1 + y^2}$$
$$\text{or , } \frac{dx}{dy} = \frac{\tan^{-1} y}{1 + y^2} - \frac{1}{1 + y^2} \cdot x$$
$$\text{or , } \frac{dx}{dy} + \frac{1}{1 + y^2} \cdot x = \frac{\tan^{-1} y}{1 + y^2} \quad ....(1)$$

which is a linear diff. equation of the form

$\frac{dx}{dy} + Px = Q$, where $P = \frac{1}{1 + y^2}$, $Q = \frac{\tan^{-1} y}{1 + y^2}$

यह $\frac{dx}{dy} + Px = Q$ **के रूप का रैखिक अवकल समीकरण है, जहाँ** $P = \frac{1}{1 + y^2}$, $Q = \frac{\tan^{-1} y}{1 + y^2}$

$$\therefore \text{ I.F.} = e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

$\therefore$ Solution of the differential equation (1) will be

अतः अवकल समीकरण (1) का हल होगा : -

$$x \cdot e^{\tan^{-1} y} = \int e^{\tan^{-1} y} \cdot \frac{\tan^{-1} y}{1 + y^2} dy + c$$
$$= \int te^t dt + c \ , \quad \text{where} \quad \tan^{-1} y = t$$
$$= t\int e^t dt - \int \left\{ \frac{d(t)}{dt} . \int e^t dt \right\} dt + c$$
$$= te^t - \int e^t dt + c$$
$$= te^t - e^t + c$$
$$= e^t(t - 1) + c$$
$$\therefore \quad x \cdot e^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + c$$
$$\therefore \quad x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

10 Ans : -

दिया गया अवकल समीकरण है

Given differential equation is

$$\frac{dy}{dx} + xy = xy^3 \quad ....(1)$$
$$\text{or , } \frac{1}{y^3} \cdot \frac{dy}{dx} + \frac{xy}{y^3} = x$$
$$\text{or , } \frac{1}{y^3}\frac{dy}{dx} + \frac{1}{y^2} \cdot x = x \quad ....(2)$$

Put $\frac{1}{y^2} = t \qquad \therefore -\frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$

$$\text{or , } \frac{1}{y^3}\frac{dy}{dx} = -\frac{1}{2}\frac{dt}{dx} \quad ....(3)$$

$\therefore$ Equation (2) becomes

$$-\frac{1}{2}\frac{dt}{dx} + t.x = x$$
$$\text{or , } \frac{dt}{dx} - 2x \cdot t = -2x \quad ....(4)$$

which is a linear differential equation of the form

$\frac{dt}{dx} + Pt = Q$, where $P = -2x$ and $Q = -2x$

यह $\frac{dt}{dx} + Pt = Q$ **के रूप का रैखिक अवकल समीकरण है जहाँ** $P = -2x$ and $Q = -2x$

$$\therefore \text{ I.F.} = e^{\int P dx} = e^{\int -2x dx} = e^{-x^2}$$

$\therefore$ Solution of equation (4) will be

अतः समीकरण (4) का हल होगा : -

$$t.e^{-x^2} = \int e^{-x^2}(-2x)dx + c$$
$$= e^{-x^2} + c$$
$$\Rightarrow \frac{1}{y^2} \cdot e^{-x^2} = e^{-x^2} + c$$
$$\Rightarrow \frac{1}{y^2} = 1 + ce^{x^2}$$

Which is the required general solution.

यही अभीष्ट व्यापक हल है।

Chapter: 10 Vector Algebra सदिश बीजगणित

बहुविकल्पीय प्रश्न (MCQ)

1 Marks Question:-

1. Position vector of the point (x,y,z) is

बिन्दु (x,y,z)का स्थिति सदिश है।

(A) $x\hat{i}-y\hat{j}-z\hat{k}$ (B) $x\hat{i}+y\hat{j}-z\hat{k}$

(C) $x\hat{i}-y\hat{j}+z\hat{k}$ (D) $x\hat{i}+y\hat{j}+z\hat{k}$

2. Position vector of the point (1,0,2) is

बिन्दु (1,0,2) का स्थिति सदिश है।

(A) $\hat{i}+\hat{j}+2\hat{k}$ (B) $\hat{i}+2\hat{j}$

(C) $\hat{i}+3\hat{k}$ (D) $\hat{i}+2\hat{k}$

3. Find the maginitude of vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

सदिश $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ का परिमाण होगा।

(A) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ **(B)** $\sqrt{3}$

(C) 3 **(D)** $\frac{1}{\sqrt{3}}$

4. Magnitude of vector $2\hat{i}-7\hat{j}-3\hat{k}$ is

सदिश $2\hat{i}-7\hat{j}-3\hat{k}$ का परिमाण है।

(A) $\sqrt{61}$ **(B)** $\sqrt{62}$

(C) $\sqrt{64}$ **(D)** $\sqrt{32}$

5. If vector $2\hat{i}+3\hat{j}=x\hat{i}+y\hat{j}$ then x = ___

यदि सदिश $2\hat{i}+3\hat{j}=x\hat{i}+y\hat{j}$ हो तो x = ___

(A) 2 (B) 3

(C) 4 (D) 9

6. The projection of vector $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ on $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ is

सदिश $\vec{a}=2\hat{i}+3\hat{j}+2\hat{k}$ का सदिश $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$ पर प्रक्षेप होगा।

(A) $\frac{10}{\sqrt{6}}$ **(B)** $\frac{15}{\sqrt{6}}$

(C) $\frac{5\sqrt{6}}{3}$ **(D)** $\frac{6\sqrt{5}}{3}$

7. The projection of vector $\hat{i}+\hat{j}$ on $\hat{i}-\hat{j}$ is

सदिश $\hat{i}+\hat{j}$ का सदिश $\hat{i}-\hat{j}$ पर प्रक्षेप होगा।

(A) 2 **(B) 0**

(C) -1 **(D) 1**

8. If $\vec{a}=5\hat{i}+7\hat{J}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}-\hat{k}$ then $\vec{a}-\vec{b}$ is

यदि $\vec{a}=5\hat{i}+7\hat{J}-3\hat{k}$ और $\vec{b}=2\hat{i}-3\hat{j}-\hat{k}$ तो $\vec{a}-\vec{b}$ होगा।

(A) $7\hat{i}+10\hat{j}-4\hat{k}$ **(B)** $3\hat{i}+10\hat{j}-2\hat{k}$

(C) $7\hat{i}+4\hat{j}-2\hat{k}$ **(D)** $10\hat{i}-21\hat{j}+3\hat{k}$

9. If $\vec{A}=2\hat{i}-3\hat{j}$ and $\vec{B}=-\hat{i}-\hat{j}$ then $\vec{A}+\vec{B}$ is equal to

यदि $\vec{A}=2\hat{i}-3\hat{j}$ और $\vec{B}=-\hat{i}-\hat{j}$ है तो $\vec{A}+\vec{B}$ = ?

(A) $3\hat{i}-4\hat{j}$ **(B)** $3\hat{i}-2\hat{j}$

(C) $\hat{i}-4\hat{j}$ **(D)** $2\hat{i}-3\hat{j}$

10. Find the unit vector in the direction of $\overrightarrow{AB}$ where $A(1,2,3)$ and $B(4,5,6)$ are the given point

सदिश $\overrightarrow{AB}$ के अनुदिश मात्रक सदिश ज्ञात कीजिए जहाँ $A(1,2,3)$ और $B(4,5,6)$ है।

(A) $5i+7\hat{j}+9\hat{k}$ **(B)** $3\hat{i}+3\hat{j}+3\hat{k}$

(C) $4\hat{i}+10\hat{J}+18\hat{K}$ **(D)** $-3\hat{i}+3\hat{j}-3\hat{k}$

11. If $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ and $\vec{b}=4\hat{i}-2\hat{j}+2\hat{k}$ are perpendicular then $\lambda=?$

यदि $\vec{a}=2\hat{i}+\lambda\hat{j}+\hat{k}$ और $\vec{b}=4\hat{i}-2\hat{j}+2\hat{k}$ लम्बवत्त हो, तो $\lambda=?$

(A) 5 (B) 6

(C) 4 (D) -5

12. The scalar product of $5\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}-4\hat{j}+7\hat{k}$ is

सदिश $5\hat{i}+\hat{j}-3\hat{k}$ और $3\hat{i}-4\hat{j}+7\hat{k}$ का सदिश गुणक होगा।

(A) 15 **(B) -15**

(C) 10 **(D) -10**

13. Which vector direction is along the vector $\hat{i}+2\hat{j}+3\hat{k}$?

सदिश $\hat{i}+2\hat{j}+3\hat{k}$ के अनुदिश सदिश कौन है?

(A) $2\hat{i}-4\hat{j}+6\hat{k}$ **(B)** $\hat{i}-2\hat{j}+3\hat{k}$

(C) $2\hat{i}+4\hat{j}+6\hat{k}$ **(D)** $3(\hat{i}-2\hat{j}-2\hat{k})$

14. If $\vec{a}$ and $\vec{b}$ are non.zero vector and $\vec{a}.\vec{b}=0$ then

यदि $\vec{a}$ और $\vec{b}$ अशून्य सदिश और $\vec{a}.\vec{b}=0$ है तो

(A) $\vec{a}=\vec{b}$ (B) $\vec{a}\parallel\vec{b}$
(C) $|\vec{a}|=|\vec{b}|$ (D) $\vec{a}\perp\vec{b}$

15. The Addative inverse of vector $-2\hat{i}+\hat{j}-\hat{k}$ is

सदिश $-2\hat{i}+\hat{j}-\hat{k}$ का योज्य प्रतिलोम (ऋणात्मक) मान है।

(A) $-2\hat{i}-\hat{j}-\hat{k}$ (B) $\frac{\hat{i}}{2}+\hat{j}-\hat{k}$
(C) $\frac{1}{6}(-2\hat{i}+6-\hat{k})$ (D) $2\hat{i}-\hat{j}+\hat{k}$

16. If $|\vec{a}|=1, \vec{b}|=2$ and $\vec{a}.\vec{b}=1$ then angle between $\vec{a}$ and $\vec{b}$ is

यदि $|\vec{a}|=1, \vec{b}|=2$ और $\vec{a}.\vec{b}=1$ हो तो $\vec{a}$ और $\vec{b}$ के बीच का कोण होगा।

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ (D) $\frac{2\pi}{3}$

17. The unit vector along the vector

$\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$ is

सदिश $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$ के अनुदिश मात्रक सदिश होगा।

(A) $\frac{1}{\sqrt{5}}(2\hat{i}+3\hat{j}+\hat{k})$ (B) $\frac{1}{\sqrt{6}}(2\hat{i}+3\hat{j}+\hat{k})$
(C) $\frac{1}{\sqrt{14}}(2\hat{i}+3\hat{j}+\hat{k})$ (D) $\frac{1}{14}(2\hat{i}+3\hat{j}+\hat{k})$

18. If $|\vec{a}|=3, |\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector then angle between $\vec{a}$ and $\vec{b}$ is

यदि $|\vec{a}|=3, |\vec{b}|=\frac{\sqrt{2}}{3}$ और $\vec{a}\times\vec{b}$ एक ईकाई सदिश हो तो सदिश $\vec{a}$ और $\vec{b}$ के बीच का कोण होगा।

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

19. If $|\vec{a}|\neq 0, |\vec{b}|\neq 0$ and $\vec{a}\times\vec{b}=0$ then

यदि $|\vec{a}|\neq 0, |\vec{b}|\neq 0$ और $\vec{a}\times\vec{b}=0$ हो तो

(A) $\vec{a}=\vec{b}$ (B) $\vec{a}\perp\vec{b}$
(C) $\vec{a}\parallel\vec{b}$ (D) $\vec{a}.\vec{b}=\vec{a}\times\vec{b}$

20. If $|\vec{a}|=2, |\vec{b}|=3$ and $\vec{a}.\vec{b}=4$ then $|\vec{a}-\vec{b}|=?$

यदि $|\vec{a}|=2, |\vec{b}|=3$ और $\vec{a}.\vec{b}=4$ है तो $|\vec{a}-\vec{b}|=?$

(A) $\sqrt{5}$ (B) 5
(C) $\frac{1}{5}$ (D) $\frac{1}{\sqrt{5}}$

21. If θ is the angle between $\vec{a}$ and $\vec{b}$ such that $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ then θ is

यदि सदिश $\vec{a}$ और $\vec{b}$ के बीच का कोण θ है और जहाँ $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$ हो तो θ का मान होगा।

(A) 0 (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ (D) π

22. The direction cosine of the vector $\hat{i}+2\hat{j}+3\hat{k}$ is

सदिश $\hat{i}+2\hat{j}+3\hat{k}$ का दिक् –कोसाइन होगा।

(A) 1,2,3 (B) $\frac{\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{14}}$
(C) $\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{3}{\sqrt{14}}$ (D) $\frac{-1}{\sqrt{14}},\frac{-2}{\sqrt{14}},\frac{-3}{\sqrt{14}}$

VERY SHORT QUESTIONS

1. Find the value of x, y and z, where vector $\vec{a}=x\hat{i}+2\hat{j}+z\hat{k}$ and $\vec{b}=2\hat{i}+y\hat{j}+\hat{k}$ are equal.

x,y और z का मान ज्ञात कीजिए, जहाँ सदिश $\vec{a}=x\hat{i}+2\hat{j}+z\hat{k}$ और $\vec{b}=2\hat{i}+y\hat{j}+\hat{k}$ बराबर है।

2. Find a vector in the direction of the vector $5\hat{i}-\hat{j}+2\hat{k}$. which has magnitude 8 units.

सदिश $5\hat{i}-\hat{j}+2\hat{k}$ के अनुदिश सदिश ज्ञात कीजिए , जिसका परिमाण 8 इकाई है।

3. If $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$ then find a unit vector in the direction of $(\vec{a}+\vec{b})$.

यदि $\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$ और $\vec{b}=-\hat{i}+\hat{j}-\hat{k}$ हो तो सदिश $(\vec{a}+\vec{b})$ के अनुदिश इकाई सदिश ज्ञात कीजिए।

4. If A(1,2,-3) and B(-1,2,1) are two given point in the space then find $\overrightarrow{AB}$.

यदि बिन्दु A(1,2,-3) और बिन्दु B(-1,2,1) अंतरिक्ष पर स्थित हो तो सदिश $\overrightarrow{AB}$ ज्ञात करें।

5. Find the position vector of a point R which divides the line joining point P(1,2,-1) and Q(-1,1,1) in the ratio 2:1 internally.

बिन्दु P(1,2,-1) और Q(-1,1,1) को मिलाने वाली रेखा को बिन्दु R, 2:1 के अनुपात में अन्तः विभाजित

करती हो तो बिन्दु R का बिन्दु सदिश क्या होगा ?

6. **Find the value of P for which $P(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.**

P का मान ज्ञात करे जहाँ $P(\hat{i}+\hat{j}+\hat{k})$ एक इकाई सदिश है।

7. **Find the angle between the vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$**

सदिश $\hat{i}-2\hat{j}+3\hat{k}$ और $3\hat{i}-2\hat{j}+\hat{k}$ के बीच का कोण ज्ञात कीजिए।

8. **If $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$ then show that vector $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular.**

यदि $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ और $\vec{b}=\hat{i}+3\hat{j}-5\hat{k}$ तो सिद्व करें कि सदिश $\vec{a}+\vec{b}$ और $\vec{a}-\vec{b}$ लम्बवत् है।

9. **If $\vec{a}$ is unit vector and $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$ then Find the value of $|\vec{x}|$**

यदि $\vec{a}$ इकाई सदिश और $(\vec{x}-\vec{a}).(\vec{x}+\vec{a})=12$ हो तो $|\vec{x}|$ का मान ज्ञात कीजिए।

10. **If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$ then Find the value of $|\vec{a}\times\vec{b}|$**

यदि $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}$ और $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$ हो तो $|\vec{a}\times\vec{b}|$ ज्ञात कीजिए।

11. **If $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$ then Find the value of λ and μ.**

यदि $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$ तो λ और μ को ज्ञात कीजिए।

12. **Find the area of triangle whose vertices are A(1,1,2), B(2,3,5) and C(1,5,5).**

एक त्रिभुज का क्षेत्रफल ज्ञात कीजिए, जिसके शीर्ष **A(1,1,2), B(2,3,5)** और **C(1,5,5)** हैं।

13. **Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear.**

दर्शाइए कि बिन्दु $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k})$ और $C(7\hat{i}-\hat{k})$ संरेख है।

14. **Find the area of parallelogram whose adjcent sides are represented by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$**

एक समांतर चतुर्भुज का क्षेत्रफल ज्ञात कीजिए जिसकी संलग्न भुजाएँ $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ और $\vec{b}=2\hat{i}-7\hat{j}+\hat{k}$ द्वारा निर्धारित है।

15. **Find the projection of $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$**

सदिश $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ का सदिश $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$ पर प्रक्षेप ज्ञात करे।

16. **Find value (मान ज्ञात करें।)**

$\hat{i}.(\hat{j}\times\hat{k})+\hat{j}.(\hat{i}\times\hat{k})+\hat{k}.(\hat{i}\times\hat{j})$

3 Marks Question (Vector)

1. **Show that the point A, B and C whose position vectors $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ respectively, are vertices of right angle triangle.**

दर्शाइए कि बिन्दु **A, B** और **C** जिनके स्थिति सदिश क्रमशः $\vec{a}=3\hat{i}-4\hat{j}-4\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}$ और $\vec{c}=\hat{i}-3\hat{j}-5\hat{k}$ है, एक समकोण त्रिभुज के शीर्षो का निर्माण करता है।

2. **If $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$ such that $(\vec{a}+\lambda\vec{b})\perp\vec{c}$ then Find the value of λ.**

यदि $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ और $\vec{c}=3\hat{i}+\hat{j}$ इस प्रकार है कि $(\vec{a}+\lambda\vec{b})\perp\vec{c}$ तो λ का मान ज्ञात कीजिए।

3. **If A(1,2,3), B(-1,0,0) and C(0,1,2) be the vertices of ΔABC then find the value ∠ABC.**

यदि किसी **ΔABC** के शीर्ष **A,B,C** क्रमशः **(1,2,3), (-1,0,0)** और **(0,1,2)** है तो **∠ABC** का मान ज्ञात कीजिए।

4. **If $\vec{a},\vec{b},\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ Find the angle between $\vec{a}$ and $\vec{b}$.**

यदि $\vec{a},\vec{b},\vec{c}$ तीन सदिश इस प्रकार है कि $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ और $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$ है तो सदिश $\vec{a}$ और $\vec{b}$ के बीच कोण का मान ज्ञात करें।

5. **If $\vec{a},\vec{b},\vec{c}$ are vectors such that $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ and each vector is perpendicular to the sum of the other two, then find $|\vec{a}+\vec{b}+\vec{c}|$.**

यदि $\vec{a},\vec{b},\vec{c}$ सदिश इस प्रकार है कि $|\vec{a}|=5, |\vec{b}|=4, |\vec{c}|=3$ और प्रत्येक सदिश अन्य दो सदिशो के योग पर लम्बवत् है तो $|\vec{a}+\vec{b}+\vec{c}|$ का मान ज्ञात करें।

6. **Show that the points A(1,2,7), B(2,6,3) and C(3,10,-1) are collinear.**

दर्शाइए की बिन्दु A(1,2,7), B(2,6,3) और C(3,10,-1) संरेख है।

(MCQ) Answer, Chapter - Vector

1. D	9. C	17. C
2. B	10. B	18. B
3. B	11. A	19. C
4. B	12. D	20. A
5. A	13. C	21. B
6. A	14. D	22. C
7. B	15. D	
8. B	16. A	

Very Short Questions Answer

1. Given that $\vec{a}=\vec{b}$

$$\Rightarrow (x\hat{i}+2\hat{j}+z\hat{k})=(2\hat{i}+y\hat{j}+\hat{k})$$

so that, the component of $\hat{i},\hat{j}$ and $\hat{k}$ are equal.

$\therefore x=2, y=2, z=1$

2. *Let* $\vec{a}=5\hat{i}-\hat{j}+2\hat{k}$

Then $|\vec{a}|=\sqrt{5^2+(-1)^2+2^2}$

$=\sqrt{25+1+4}$

$=\sqrt{30}$

$\therefore$ *unit vector,*

$$\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\left\{\frac{5}{\sqrt{30}}\hat{i}-\frac{1}{\sqrt{30}}\hat{j}+\frac{2}{\sqrt{30}}\hat{k}\right\}$$

Hence, the required vector $=8\hat{a}$

$$=8\left\{\frac{5}{\sqrt{30}}\hat{i}-\frac{1}{\sqrt{30}}\hat{j}+\frac{2}{\sqrt{30}}\hat{k}\right\}$$

3. *Given that*

$\vec{a}=2\hat{i}-\hat{j}+2\hat{k}$

$b=-\hat{i}+\hat{j}-\hat{k}$

$\therefore \vec{a}+\vec{b}=(2-1)\hat{i}+(-1+1)\hat{j}+(2-1)\hat{k}$

$i+0\hat{j}+\hat{k}=\hat{i}+\hat{k}$

$|\vec{a}+\vec{b}|=\sqrt{1^2+1^2}=\sqrt{2}$

Unit vector along $(\vec{a}+\vec{b})$

$$=\frac{(\vec{a}+\vec{b})}{|\vec{a}+\vec{b}|}=\frac{1}{2}(\hat{i}+\hat{k})$$

4. Let O be the origin of the space.

then $\overrightarrow{OA}=\hat{i}+2\hat{j}-3\hat{k}$

& $\overrightarrow{OB}=-\hat{i}+2\hat{j}+\hat{k}$

$\therefore \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$

$=(-\hat{i}+2\hat{j}+\hat{k})-(\hat{i}+2\hat{j}-3\hat{k})$

$=(-1-1)\hat{i}+(2\hat{j}-2\hat{j})+(1+3)\hat{k}$

$=-2\hat{i}+0\hat{j}+4\hat{k}=-2\hat{i}+4\hat{k}$

5. **Let O be the origin**

so that $\overrightarrow{OP}=\hat{i}+2\hat{j}-\hat{k}=\vec{a}(\text{Let})$

$\overrightarrow{OQ}=-\hat{i}+\hat{j}+\hat{k}=\vec{b}(\text{Let})$

Point R divides PQ internally in the ratio 2:1

then $\overrightarrow{OR}=\dfrac{m\vec{b}+n\vec{a}}{m+n}\ [\because m:n=2:1]$

$$=\frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1}$$

$$=\frac{(-2+1)\hat{i}+(2+2)\hat{j}+(2-1)\hat{k}}{3}$$

$$=\frac{-\hat{i}+4\hat{j}+\hat{k}}{3}$$

Hence, the position vector of R is $\dfrac{-\hat{i}+4\hat{j}+\hat{k}}{3}$.

6. *Let* $\vec{a}=P(\hat{i}+\hat{j}+\hat{k})$

$|\vec{a}|=\sqrt{p^2+p^2+p^2}=\pm P\sqrt{3}$

We have given that $|\vec{a}|=1$

$\Rightarrow \pm P\sqrt{3}=1$

$P=\pm\dfrac{1}{\sqrt{3}}$

7.

Let $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$

$\vec{b}=3\hat{i}-2\hat{j}+\hat{k}$

Now $|\vec{a}|=\sqrt{1^2+(-2)^2+3^2}=\sqrt{1+4+9}=\sqrt{14}$

$|\vec{b}|=\sqrt{3^2+(-2)^2+1^2}=\sqrt{9+4+1}=\sqrt{14}$

$\vec{a}.\vec{b}=(\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-2\hat{j}+\hat{k})$

$=3+4+3=10$

Let θ *be the angle between* $\vec{a}$ *and* $\vec{b}$,

then $\cos\theta=\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\dfrac{10}{\sqrt{14}.\sqrt{14}}=\dfrac{10^5}{14_7}$

$\cos\theta=\dfrac{5}{7}$

$\therefore \theta=\cos^{-1}\dfrac{5}{7}$

8.

Given that

$\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$

Now $\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{k}$

$\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{k}$

$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (6\hat{i} + 2\hat{j} - 8\hat{k}).(4\hat{i} - 4\hat{j} + 2\hat{k})$

$= 24 - 8 - 16$

$= 0$

Hence, $(\vec{a} + \vec{b})$ *and* $(\vec{a} - \vec{b})$ *is perpendicular to each other.*

9.

$(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$

$\Rightarrow \vec{x}.\vec{x} + \vec{x}.\vec{a} - \vec{a}.\vec{x} - \vec{a}.\vec{a} = 12$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 12$

$\Rightarrow |\vec{x}|^2 - 1^2 = 12 \left[\begin{array}{l} \because \vec{a} \text{ is unit vector} \\ \therefore |\vec{a}| = 1 \end{array}\right]$

$|\vec{x}|^2 = 13$

$\therefore |\vec{x}| = \sqrt{13}$

10.

Given that $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$$

$= (-2 - 15)\hat{i} - (-4 - 9)\hat{j} + (10 - 3)\hat{k}$

$= -17\hat{i} + 13\hat{j} + 7\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + (13)^2 + 7^2}$

$= \sqrt{289 + 169 + 49}$

$= \sqrt{507}$

11.

Given that

$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$

$$\text{Now} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = (2\lambda - 6)\hat{i} + (27 - 2\mu)\hat{j} + (2\lambda - 6)\hat{k}$$

$\because (2\lambda - 6)\hat{i} + (27 - 2\mu)\hat{j} + (2\lambda - 6)\hat{k} = \vec{0}$

Equating the coefficient of $\hat{i}, \hat{j}$ & $\hat{k}$. *We, get*

$2\lambda - 6 = 0 \Rightarrow \lambda = 3$

$27 - 2\mu = 0 \Rightarrow \mu = \frac{27}{2}$

12.

Let O be the origin

then $\overrightarrow{OA} = \hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 0\hat{i} + 4\hat{j} + 3\hat{k}$

$\therefore$ *area of* $\triangle ABC = \left| \frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) \right|$

$$\text{Now } \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$= (6 - 12)\hat{i} + (0 - 3)\hat{j} + (4 - 0)\hat{k}$

$= -6\hat{i} - 3\hat{j} + 4\hat{k}$

Now $\frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) = 2\hat{i} - \frac{3}{2}\hat{j} - 3\hat{k}$

Area of $\triangle ABC = \left| \frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) \right| = \sqrt{9 + \frac{9}{4} + 4}$

$= \sqrt{\frac{61}{4}} = \frac{1}{2}\sqrt{61}$ *Sq. Unit*

13. *Let O be the origin. then*

$\overrightarrow{OA} = -2\hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{OB} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{OC} = 7\hat{i} - \hat{k}$

Now $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (\hat{i} + 2\hat{j} + 3\hat{k}) - (-2\hat{i} + 3\hat{j} + 5\hat{k})$

$= (1 + 2)\hat{i} + (2 - 3)\hat{j} + (3 - 5)\hat{k}$

$= 3i - \hat{j} - 2\hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= (7i - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= 6\hat{i} - 2\hat{j} - 4\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (7\hat{i} - \hat{k}) - (-2\hat{i} + 3\hat{j} + 5\hat{k})$

$= 9\hat{i} - 3\hat{j} - 6\hat{k}$

$$\text{Now } \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -2 \\ 9 & -3 & -6 \end{vmatrix}$$

$= (6 - 6)\hat{i} + (-18 + 18)\hat{j} + (-9 + 9)\hat{k}$

$= 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$

Hence, point A, B and C are collinears.

14.

Given that adjacent sides of Parallelogram are

$\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

Then the area of parallelogram $= |\vec{a} \times \vec{b}|$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$

$= (-1+21)\hat{i} + (6-1)\hat{j} + (-7+3)\hat{k}$

$= 20\hat{i} + 5\hat{j} - 4\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(20)^2 + 5^2 + (-4)^2}$

$= \sqrt{400 + 25 + 16}$

$= \sqrt{441} = 21$

Hence, area of parallelogram is 21 *Sq. Unit.*

15.

Projection of $\vec{a}$ *on* $\vec{b} = \dfrac{\vec{a}.\vec{b}}{|\vec{b}|}$

$= \dfrac{(2\hat{i} - \hat{j} + \hat{k}).(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{(1)^2 + (-2)^2 + 1^2}}$

$= \dfrac{2+2+1}{\sqrt{1+4+1}} = \dfrac{5}{\sqrt{6}} = \dfrac{5\sqrt{6}}{6}$

16. $\hat{i}.(\hat{j} \times \hat{k}) + \hat{j}.(\hat{i} \times \hat{k}) + \hat{k}.(i \times \hat{j})$

$= \hat{i}.\hat{i} + \hat{j}.(-\hat{j}) + \hat{k}.\hat{k}$ $\quad \left[\begin{array}{l} \because \hat{i} \times \hat{j} = \hat{k} \\ \hat{j} \times \hat{k} = \hat{i} \\ \hat{i} \times \hat{k} = -\hat{j} \end{array}\right]$

$= 1 - 1 + 1$

$= 1$

3 Marks (Vector) Questions Answer

1. *Let O be the origin.*

Then $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$

$|\overrightarrow{AB}| = \sqrt{1+9+25} = \sqrt{35}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$

$|\overrightarrow{BC}| = \sqrt{1+4+36} = \sqrt{41}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -2\hat{i} + \hat{j} - \hat{k}$

$|\overrightarrow{AC}| = \sqrt{4+1+1} = \sqrt{6}$

Now $|\overrightarrow{BC}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{AC}|^2$

Hence the point A, B and C

are the vertices of the right angle triangle.

2.

Given that

$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{c} = 3\hat{i} + \hat{j}$

$\because (\vec{a} + \lambda\vec{b}) \perp \vec{c} \Leftrightarrow (\vec{a} + \lambda\vec{b}).\vec{c} = 0$

$\Rightarrow [(2\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + 2\hat{j} + \hat{k})].(3\hat{i} + \hat{j}) = 0$

$\Rightarrow [(2-\lambda)\hat{i} + (2+2\lambda)\hat{j} + (3+\lambda)\hat{k}].(3\hat{i} + \hat{j}) = 0$

$\Rightarrow 3(2-\lambda) + 2 + 2\lambda = 0$

$\Rightarrow 6 - 3\lambda + 2 + 2\lambda = 0$

$\Rightarrow 8 - \lambda = 0$

$\therefore \lambda = 8$

3. *Let O be the origin. then*

$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{OB} = -\hat{i}$

$\overrightarrow{OC} = \hat{j} + 2\hat{k}$

$\because \angle ABC$ *makes between* $\overrightarrow{AB}$ & $\overrightarrow{BC}$

$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= -\hat{i} - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= -2\hat{i} - 2\hat{j} - 3\hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \hat{j} + 2\hat{k} + \hat{i}$

$= \hat{i} + \hat{j} + 2\hat{k}$

$\therefore |\overrightarrow{AB}.\overrightarrow{BC}| = |-2-2-6| = 10$

$|\overrightarrow{AB}| = \sqrt{4+4+9} = \sqrt{17}$

$|\overrightarrow{BC}| = \sqrt{1+1+4} = \sqrt{6}$

$\therefore \cos(\angle ABC) = \dfrac{\overrightarrow{AB}.\overrightarrow{BC}}{|\overrightarrow{AB}||\overrightarrow{BC}|}$

$= \dfrac{10}{\sqrt{17}.\sqrt{6}}$

$\therefore \angle ABC = \cos^{-1}\left(\dfrac{10}{\sqrt{102}}\right)$

4.

Let θ *be the angle between* $\vec{a}$ *and* $\vec{b}$

Now

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow (\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = (-\vec{c}).(-\vec{c})$

$$\Rightarrow \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}=\vec{c}.\vec{c}$$
$$\Rightarrow |\vec{a}|^2+2\vec{a}.\vec{b}+|\vec{b}|^2=|\vec{c}|^2$$
$$[\because \vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta]$$
$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=|\vec{c}|^2$$
$$\Rightarrow 3^2+5^2+2.3.5\cos\theta=7^2$$
$$\Rightarrow 9+25+30\cos\theta=49$$
$$\Rightarrow 30\cos\theta=49-34=15$$
$$\Rightarrow \cos\theta=\frac{15}{30}=\frac{1}{2}$$
$$\therefore \theta=\cos^{-1}\frac{1}{2}$$
$$\theta=\cos^{-1}(\cos 60^\circ)$$
$$\therefore \theta=60^\circ$$

5. *We have given that*

$\vec{a}\perp(\vec{b}+\vec{c}), \vec{b}\perp(\vec{c}+\vec{a})$ *and* $\vec{c}\perp(\vec{a}+\vec{b})$

$\Rightarrow \vec{a}.(\vec{b}+\vec{c})=0, \vec{b}.(\vec{c}+\vec{a})=0$

and $\vec{c}.(\vec{a}+\vec{b})=0$

$\Rightarrow \vec{a}.\vec{b}+\vec{a}.\vec{c}=0, \vec{b}.\vec{c}+\vec{b}.\vec{a}=0$

and $\vec{c}.\vec{a}+\vec{c}.\vec{b}=0$

By adding all, we get

$2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})=0$

$\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$

Now

$|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c}).(\vec{a}+\vec{b}+\vec{c})$

$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$

$=5^2+4^2+3^2+0 \quad (\because \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0)$

$=25+16+9$

$|\vec{a}+\vec{b}+\vec{c}|^2=50$

$|\vec{a}+\vec{b}+\vec{c}|=\sqrt{50}=5\sqrt{2}$

6.

Let O be the origin. Then

$\overrightarrow{OA}=\hat{i}+2\hat{j}+7\hat{k},$

$\overrightarrow{OB}=2\hat{i}+6\hat{j}+3\hat{k}$

$\overrightarrow{OC}=3\hat{i}+10\hat{j}-\hat{k}$

$\therefore \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OC}=(2\hat{i}+6\hat{j}+3\hat{k})-(3\hat{i}+10\hat{j}-\hat{k})$

$=-\hat{i}-4\hat{j}+4\hat{k}$

$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=(3\hat{i}+10\hat{j}-\hat{k})-(2\hat{i}+6\hat{j}+3\hat{k})$

$=\hat{i}+4\hat{j}-4\hat{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=(3\hat{i}+10\hat{j}-\hat{k})-(\hat{i}+2\hat{j}+7\hat{k})$

$=2\hat{i}+8\hat{j}-8\hat{k}$

$\therefore |\overrightarrow{AB}|=\sqrt{1+16+16}=\sqrt{33}$

$|\overrightarrow{BC}|=\sqrt{1+16+16}=\sqrt{33}$

$|\overrightarrow{AC}|=\sqrt{4+64+64}=\sqrt{4\times 33}=2\sqrt{33}$

$\because |\overrightarrow{AC}|=|\overrightarrow{AB}|+|\overrightarrow{BC}|$

Hence, points A, B and C are Collinear.

Chapter-11	Three- Dimensional Geometry त्रि-विमीय ज्यामिती

MCQ:- (बहुविकल्पीय प्रश्न) -

Q 1 **If a line makes angles $90^\circ, 60^\circ$ and 30° with the +ve direction of the axises x,y,and z respectively. then find the direction cosine .**

यदि एक रेखा x,y और z अक्षो की धनात्मक दिशा के साथ क्रमशः $90^\circ, 60^\circ$ तथा 30° का कोण बनाती है तो दिक्-कोसाइन ज्ञात करें।

(a) $0, \frac{1}{2}, \frac{\sqrt{3}}{2}$ (b) $1, \frac{\sqrt{3}}{2}, \frac{1}{2}$

(c) 0 , 0 , 1 (d) $1, \frac{1}{\sqrt{2}}, \frac{1}{2}$

Q 2 **A line makes equal angles with the axes. then the direction cosines are-**

एक रेखा की दिक्-कोसाइन क्या होगी? जो अक्षों के साथ समान कोण बनाती है।

(a) $\pm 1, \pm 1, \pm 1$

(b) $\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}$

(c) 0 , 0 , 0

(d) $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}$

Q 3 **The direction cosines of a line whose direction ratios are 2, -6, 3.**

यदि एक रेखा के दिक्-अनुपात 2,-6,3 हो तो इसकी दिक्-कोसाइन क्या होगा ?

(a) $\frac{2}{\sqrt{5}}, \frac{-6}{\sqrt{5}}, \frac{3}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{3}}, \frac{-6}{\sqrt{3}}, \frac{3}{\sqrt{3}}$

(c) $\frac{2}{7}, \frac{-6}{7}, \frac{3}{7}$ (d) $\frac{2}{49}, \frac{-6}{49}, \frac{3}{49}$

Q 4 **The direction cosines of the line segment joining the point A(-2,4,5) and B(1,2,3) are**

बिन्दूओं A (-2,4,5) और B(1,2,3) को मिलाने वाली रेखा की दिक्-कोसाइन होगा।

(a) $\frac{-3}{77}, \frac{2}{77}, \frac{-8}{77}$ (b) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

(c) $\frac{-2}{\sqrt{77}}, \frac{4}{\sqrt{77}}, \frac{-5}{\sqrt{77}}$ (d) $\frac{1}{77}, \frac{2}{77}, \frac{3}{77}$

Q 5 **The direction ratio of the vector $2\hat{i} + \hat{j} - 2\hat{k}$ are**

सदिश $2\hat{i} + \hat{j} - 2\hat{k}$ का दिक्-अनुपात होगा।

(a) 2 , 1 , -2 (b) $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

(c) $\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$ (d) $\frac{-2}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$

Q 6 **The direction cosine of the vector $-\hat{i} - 2\hat{j} - 2\hat{k}$ is**

सदिश $-\hat{i} - 2\hat{j} - 2\hat{k}$ का दिक्-कोसाइन होगा।

(a) $\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ (b) $-1, -2, -2$

(c) $\frac{-1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{-2}{\sqrt{3}}$ (d) $\frac{-1}{3}, \frac{-2}{3}, \frac{-2}{3}$

Q 7 **The direction cosines of the y-axis are**

y-अक्ष का दिक्-कोसाइन होगा।

(a) $(0,0,0)$ (b) $(1,0,0)$

(c) $(0,1,0)$ (d) $(0,0,1)$

Q 8 **The vector equation of the line passing through the points (-1,0,2) and (3,4,6) is :**

बिन्दू(-1,0,2) और (3,4,6) से होकर जाने वाली रेखा का सदिश समीकरण होगा।

(a) $\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(b) $\hat{i} - 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(c) $-\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$

(d) $-\hat{i} + 2\hat{k} + \lambda(4\hat{i} - 4\hat{j} - 4\hat{k})$

Q 9 **The cartesian equation of a line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ then the vector form of the equation is -**

कार्तीय समीकरण $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ का सदिश समीकरण क्या होगा ?

(a) $(-5\hat{i} + 4\hat{j} + 6\hat{k}) - \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

(b) $(-5\hat{i} + 4\hat{j} - 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

(c) $(5\hat{i} + 4\hat{j} + 6\hat{k}) - \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

(d) $(5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

Q 10 The vector equation

$\vec{r}=(-3\hat{i}+5\hat{j}-6\hat{k})+\lambda(2\hat{i}+4\hat{j}+2\hat{k})$ then the cartesian form of the equation is

सदिश समीकरण $\vec{r}=(-3\hat{i}+5\hat{j}-6\hat{k})+\lambda(2\hat{i}+4\hat{j}+2\hat{k})$ को कार्तीय समीकरण रूप होगा।

(a) $\frac{x-3}{2}=\frac{y+5}{4}=\frac{z-6}{2}$

(b) $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z+6}{2}$

(c) $\frac{x+3}{-2}=\frac{y+5}{-4}=\frac{z+6}{-2}$

(d) None of these (इनमें से कोई नहीं)

Q 11 The distance of the plane 2x-3y+6z+7=0 from the point (2,-3,-1) is

समतल 2x-3y+6z+7=0 से बिन्दु (2,-3,-1) की दूरी होगी।

(a) 4 (b) 3

(c) 2 (d) $\frac{1}{5}$

Q 12 The direction cosines of the normal to the plane 2x-3y-6z-3=0 are-

समतल 2x-3y-6z-3=0 के अभिलंब की दिक्-कोसाइन होगा।

(a) $\frac{2}{7},\frac{-3}{7},\frac{-6}{7}$

(b) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$

(c) $\frac{-2}{7},\frac{-3}{7},\frac{-6}{7}$

(d) None of these (इनमें से कोई नहीं)

Q 13 If 2x+5y-6z+3=0 be the equation of plane then the equation of any plane parallel to the given plane is

यदि 2x+5y-6z+3=0 एक समतल का समीकरण हो तो इसके समान्तर समतल का एक समीकरण होगा।

(a) 3x+5y-6z+3=0

(b) 2x-5y-6z+3=0

(c) 2x+5y-6z+k=0

(d) None of these (इनमें से कोई नहीं)

Very Short Question (2 Marks)

Q 1 Show that the lines , whose direction cosines are $\frac{12}{13},\frac{-3}{13},\frac{-4}{13}$ & $\frac{4}{13},\frac{12}{13},\frac{3}{13}$ are mutually Perpendicular.

दर्शाइए कि दिक्-कोसाइन $\frac{12}{13},\frac{-3}{13},\frac{-4}{13}$ और $\frac{4}{13},\frac{12}{13},\frac{3}{13}$ वाली रेखाएँ परस्पर लम्बवत् है।

Q 2 Show that the line segment joining the point A(1,-1,2) and B(3,4,2) is perpendicular to the line segment joining the points C(0,3,2) and D(3,5,6).

दिखाइए कि बिन्दु A(1,-1,2) और B(3,4,2) को मिलाने वाली रेखाखण्ड , बिन्दु C(0,3,2) और D(3,5,6) को मिलाने वाली रेखाखण्ड के लम्बवत् है।.

Q 3 A line passes through the point (1,2,3) and is parallel to $3\hat{i}+2\hat{j}-2\hat{k}$.Find the equation of the line in vector forms.

बिन्दु (1,2,3) से गुजरने वाली रेखा का सदिश समीकरण ज्ञात कीजिए जो सदिश $3\hat{i}+2\hat{j}-2\hat{k}$ के समान्तर है।

Q 4 Find the vector equation of the line passing through the points(3,-2,-5) and (3,-2,6).

बिन्दुओ (3,-2,-5) और (3,-2,6) से गुजरने वाली रेखा का सदिश समीकरण को ज्ञात कीजिए।

Q 5 Find the angle between the lines having direction ratios are 3,4,5 and 4,-3,5.

दिक् अनुपात 3,4,5 और 4,-3,5 के बीच का कोण ज्ञात करे।

Q 6 Find the distance of the plane 2x-3y+6z+14=0 from origin.

मूल बिन्दु से समतल 2x-3y+6z+14=0 की दूरी ज्ञात करें।

Q 7 Find the value of λ for which the lines

$$\frac{x-1}{1}=\frac{y-2}{\lambda}=\frac{z+1}{1} \text{ and } \frac{x+1}{-\lambda}=\frac{y+1}{2}=\frac{z-2}{2}$$

are perpendicular to each other.

λ का मान ज्ञात करे जिससे कि सरल रेखाएँ

$$\frac{x-1}{1}=\frac{y-2}{\lambda}=\frac{z+1}{1} \text{ और } \frac{x+1}{-\lambda}=\frac{y+1}{2}=\frac{z-2}{2}$$

परस्पर लम्ब हो।

Q 8 If α,β,γ be the angles, which a line makes with the positive direction of axes. Prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

यदि कोई रेखा धनात्मक नियामक अक्षो के साथ α,β,γ कोण बनाती हो, तो सिद्ध कीजिए कि $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

Q 9 Find the direction cosines for a line that is perpendicular to each of the two lines whose direction ratios are (2,-1,2) and (3,0,1).

उस सरल रेखा के लिए दिक्-कोसाइन ज्ञात कीजिए। जो दिक् अनुपातों (2,-1,2) और (3,0,1) वाली दो सरल रेखाओ मे से प्रत्येक पर लम्ब है।

Short Question (3 Marks)

Q 1 **Find the angle between the lines $\vec{r}=-2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ and $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$.**

दिये गये रेखाओ $\vec{r}=-2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ और $\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$ के बीच का कोण ज्ञात करें।

Q 2 **Find the angle between the lines**

$$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$$

दिये गये रेखाओं

$$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3} \text{ और } \frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$$

के बीच का कोण ज्ञात करें।

Q 3 **Find the value of P, where lines**

$$\frac{1-x}{3}=\frac{7y-14}{2P}=\frac{z-3}{2} \text{ and } \frac{7-7x}{3P}=\frac{y-5}{1}=\frac{6-z}{5}$$

are perpendicular.

P का मान ज्ञात कीजिए, जहाँ रेखाएँ

$$\frac{1-x}{3}=\frac{7y-14}{2P}=\frac{z-3}{2} \text{ और } \frac{7-7x}{3P}=\frac{y-5}{1}=\frac{6-z}{5}$$

परस्पर लम्ब हो।

Q 4 **Find the distance between the lines $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$**

रेखाओं $\vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ और $\vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(2\hat{i}+3\hat{j}+6\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 5 **Find the distance of a plane 2x-3y+4z-6=0 from the origin.**

समतल 2x-3y+4z-6=0 की मूल बिन्दु से दूरी ज्ञात कीजिए।

Q 6 **Find the angle between of the planes 2x+y-2z=5 and 3x-6y-2z=7 in vector method.**

दो समतलो 2x+y-2z=5 और 3x-6y-2z=7 के बीच का कोण सदिश विधि द्वारा ज्ञात कीजिए।

Q 7 **Find the angle between the planes x+y+2z=9 and 2x–y+z=6.**

दो समतलो x+y+2z=9 और 2x–y+z=6 के बीच का कोण ज्ञात कीजिए।

Q 8 **Find the value of λ for which the planes 2x-4y+3z=7 and x+2y+λz=18 are perpendicular to each other.**

समतलों 2x-4y+3z=7 और x+2y+λz=18 एक दूसरे के लम्बवत् हो तो λ का मान ज्ञात करें।

Q 9 **Find the equation of the plane passing through the point (1,4,-2) and parallel to the plane -2x+y-3z=0.**

उस समतल का समीकरण ज्ञात कीजिए, जो बिन्दु (1,4,-2) से होकर जाती है एवं समतल -2x+y-3z=0 के समान्तर हो।

Q 10 **Find the equation of the plane passing through the points (1,1,-1) , (6,4,-5) and (-4,-2,3).**

बिन्दुओं (1,1,-1) , (6,4,-5) और (-4,-2,3) से गुजरने वाले समतल का समीकरण ज्ञात कीजिए।

Long Question (5 Marks)

Q 1 **Find the shortest distance of lines $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$.**

रेखाओं $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ और $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 2 **Find the shortest distance of lines $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$.**

रेखाओ $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$ और $\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 3 **Find the shortest distance of lines $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ and $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$.**

रेखाओ $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ और $\vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$ के बीच की दूरी ज्ञात कीजिए।

Q 4 **Find the shortest distance between the line**

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1} \text{ and } \frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$$

रेखाओ $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ और $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ के बीच की न्यूनत्तम दूरी ज्ञात कीजिए।

Q 5 **Find the equation of the plane passing through the intersection of the plane 3x-y+2z-4=0 and x+y+z-2=0 and the point (2,2,1).**

उस समतल का समीकरण ज्ञात कीजिए, जो समतलो 3x-y+2z-4=0 और x+y+z-2=0 के प्रतिछेदन तथा बिन्दु (2,2,1) से होकर जाती है।

Q 6 **Find the equation of the plane passing through the line of intersection of the planes x+y+z=1 and 2x+3y+4z=5 which is perpendicular to the plane x-y+z=0.**

समतलो x+y+z=1 और 2x+3y+4z=5 के प्रतिच्छेदन रेखा से होकर जाने वाली तथा समतल x-y+z=0 पर लम्बवत् समतल का समीकरण ज्ञात कीजिए।

Q 7 Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ and the point (2,1,3).

उस समतल का सदिश समीकरण ज्ञात कीजिए जो समतल $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ के प्रतिच्छेदन और बिन्दु (2,1,3) से होकर जाता है।

Q 8 Find the angle between the planes, whose vector equation are $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$.

समतलो,जिनके सदिश समीकरण $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=5$ और $\vec{r}.(3\hat{i}-3\hat{j}+5\hat{k})=3$ के बीच का कोण ज्ञात करे।

Q 9 Find the equation of the plane passing through the point (1,-2,4) , (3,-4,5) and perpendicular to the plane x+y-2z=6.

उस तल का समीकरण ज्ञात कीजिए जो बिन्दुओं (1,–2,4) और (3, –4, 5) से गुजरती है तथा समतल x+y-2z=6 के लम्बवत् है।

MCQ 1-Marks Solution

Ans : -

1 - (a) 2 - (d) 3 - (c) 4 - (b) 5 - (a) 6 - (d) 7 - (c)

8 - (c) 9 - (d) 10 - (b) 11 - (c) 12 - (a) 13 - (c)

2-Marks Solution

1 Ans :

The direction cosines of first lines are

$\frac{12}{13}, -\frac{3}{13}, -\frac{4}{13}$

So that $l_1 = \frac{12}{13}, m_1 = -\frac{3}{13}, n_1 = -\frac{4}{13}$

The direction cosines of second lines are

$\frac{4}{13}, \frac{12}{13}, \frac{3}{13}$

So that $l_2 = \frac{4}{13}, m_2 = \frac{12}{13}, n_2 = \frac{3}{13}$

Now ,

$$l_1l_2+m_1m_2+n_1n_2 = \frac{12}{13}\cdot\frac{4}{13}+\frac{-3}{13}\cdot\frac{12}{13}+\frac{-4}{13}\cdot\frac{3}{13}$$
$$= \frac{48}{169}-\frac{36}{169}-\frac{12}{169}$$
$$= 0$$

Hence, the both lines are mutually perpendicular.

2 Ans : -

The direction ratios of the lines AB are

(3-1),(4+1),(-2,-2) i.e. 2,5,-4

$\therefore$ $a_1=2, b_1=5, c_1=-4$

Similarly the direction ratios of line CD are 3,2,4

$\therefore$ $a_2=3, b_2=2, c_2=4$

Now $a_1a_2+b_1b_2+c_1c_2 = 2.3+5.2+(-4).4$

$= 6+10-16$

$= 0$

Hence the line AB and CD are mutually perpendicular.

3 Ans : In vector form

The given line the passes through the point A(1,2,3) and in parallel to the vector $\vec{m} = 3\hat{i}+2\hat{j}-2\hat{k}$

the position vector of A is, $\vec{r_1} = \hat{i}+2\hat{j}+3\hat{k}$

Hence the required line is $\vec{r} = \vec{r_1}+\lambda\vec{m}$

$=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$ ....eqn(1)

In cartesian form

Taking $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

Then equation (1) becomes

$$x\hat{i}+y\hat{j}+z\hat{k} = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(3\hat{i}+2\hat{j}-2\hat{k})$$

$$\Rightarrow x\hat{i}+y\hat{j}+z\hat{k} = (1+3\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3-2\lambda)\hat{k}$$

$x = 1+3\lambda \Rightarrow \lambda = \frac{x-1}{3}$

$y = 2+2\lambda \Rightarrow \lambda = \frac{y-2}{2}$

$z = 3-2\lambda \Rightarrow \lambda = \frac{z-3}{-2}$

$\Rightarrow \frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2} = \lambda$

Hence $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$ are the required equation of the given line.

4 Ans : -

Let the position vectors of (3, -2, -5) and (3,-2,6) be $\vec{r_1}$ and $\vec{r_2}$ respectively . Then

$\vec{r_1} = 3\hat{i}-2\hat{j}-5\hat{k}$ and $\vec{r_2} = 3\hat{i}-2\hat{j}+6\hat{k}$

$\therefore \vec{r_2} - \vec{r_1} = (3\hat{i} - 2\hat{j} + 6\hat{k}) - (3\hat{i} - 2\hat{j} - 5\hat{k})$

$= 0\hat{i} + 0\hat{j} + 11\hat{k} = 11\hat{k}$

$\therefore$ The vector equation of the line is

$\vec{r} = \vec{r_1} + \lambda(\vec{r_2} - \vec{r_1})$

i.e. $\vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} - \lambda 11\hat{k}$

This is the required equation.

5 Ans : -

Let θ be the angle between the given vector

Let $a_1=3$, $b_1=4$, $c_1=5$

and $a_2=4$, $b_2=-3$, $c_2=5$

$$\therefore \cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{(\sqrt{a_1^2 + b_1^2 + c_1^2})(\sqrt{a_2^2 + b_2^2 + c_2^2})}$$

$$= \frac{3.4 + 4.(-3) + 5.5}{(\sqrt{9+16+25})(\sqrt{16+9+25})}$$

$$\cos\theta = \frac{25}{\sqrt{50} \cdot \sqrt{50}} = \frac{25}{50} = \frac{1}{2}$$

$$\theta = \cos^{-1}\frac{1}{2} = \cos(\cos 60^\circ)$$

$\therefore \quad \theta = 60^\circ$

6 Ans : -

Let O be the origin.

i.e.$x_1=0=y_1=z_1$

the d.r.s of the given plane are a=2,b=-3,c=6

So that, perpendicular distance

$$P = \left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

$$= \left| \frac{2.0 + (-3).0 + 6.0 + 14}{\sqrt{2^2 + (-3)^2 + 6^2}} \right|$$

$$= \frac{14}{\sqrt{4+9+36}}$$

$$= \frac{14}{\sqrt{49}}$$

$P = \frac{14}{7} = 2$ units.

Hence, the distance of the plane 2x–3y+6z+14=0 from origin is 2 units.

7 Ans : -

The d.r.s of the first line are $a_1=1, b_1=\lambda, c_1=1$

The d.r.s of the second line are $a_2=-\lambda, b_2=2, c_2=2$

$\because$ the lines are perpendicular to each other.

so that, $\quad a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow 1.(-\lambda) + \lambda.2 + 1.2 = 0$

$\Rightarrow \quad -\lambda + 2\lambda + 2 = 0$

$\lambda = -2$

8 Ans : -

The given line makes angles α, β, γ with the x-axis, y-axis and z-axis respectively.

So that, the direction cosine are

$l = \cos\alpha$

$m = \cos\beta$

$n = \cos\gamma$

we know that , $\quad l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

9 Ans : -

Let l,m,n be the direction cosines of the required line.

Then , $l(2) + m(-1) + n(2) = 0$

and , $\quad l(3) + m(0) + n(1) = 0$

$\Rightarrow \quad 2l - m + 2n = 0$ ....(1)

and $\quad 3l - 0m + n = 0$ ....(2)

Solve the (1) and(2) by Cross multiplying

method,we get-

$$\frac{l}{-1} = \frac{m}{6-2} = \frac{n}{3}$$

$$\Rightarrow \frac{l}{-1} = \frac{m}{4} = \frac{n}{3} = \frac{\sqrt{l^2 + m^2 + n^2}}{\sqrt{(-1)^2 + 4^2 + 3^2}} = \frac{1}{\sqrt{26}}$$

$$\therefore l = \frac{-1}{\sqrt{26}}, m = \frac{4}{\sqrt{26}}, n = \frac{3}{\sqrt{26}}$$

3-Marks Solution

1 Ans : - Let θ be the angle between the given lines .

The given lines are parallel to the vector.

$$\overrightarrow{m_1} = 3\hat{i}+2\hat{j}+6\hat{k} \text{ and } \overrightarrow{m_2} = \hat{i}+2\hat{j}+2\hat{k}$$

$$\therefore \cos\theta = \frac{\overrightarrow{m_1}.\overrightarrow{m_2}}{|\overrightarrow{m_1}||\overrightarrow{m_2}|} = \frac{(3\hat{i}+2\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{|3\hat{i}+2\hat{j}+6\hat{k}||\hat{i}+2\hat{j}+2\hat{k}|}$$

$$= \frac{3+4+12}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}$$

$$= \frac{19}{\sqrt{9+4+36}\sqrt{1+4+4}}$$

$$= \frac{19}{\sqrt{49}\sqrt{9}} = \frac{19}{7\cdot 3} = \frac{19}{21}$$

$$\cos\theta = \frac{19}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

2 Ans : -

The d.r.s. of the line $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ are 2,5,3

i.e. $a_1 = 2, b_1 = 5, c_1 = 3$

And , the d.r.s of the line $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$ are -1,8,4

i.e. $a_2 = -1, b_2 = 8, c_2 = 4$

Let θ be the angle between the given lines

$$\therefore \cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{2.(-1)+5.8+3.4}{\sqrt{4+25+9}\,.\sqrt{1+64+16}}$$

$$\cos\theta = \frac{-2+40-12}{\sqrt{38}.9} = \frac{26}{9\sqrt{38}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

3 Ans : -

The d.r.s of the line $\frac{1-x}{3} = \frac{7y-14}{2P} = \frac{z-3}{2}$

i.e. $\frac{x-1}{-3} = \frac{y-14}{\left(2P/7\right)} = \frac{z-3}{2}$ are -3, $\frac{2P}{7}$,2

The d.r.s of the line $\frac{7-7x}{3P} = \frac{y-5}{1} = \frac{6-z}{5}$

i.e. $\frac{x-1}{\left(-3P/7\right)} = \frac{y-5}{1} = \frac{z-6}{-5}$ are $-\frac{3P}{7}, 1, -5$

$\because$ the lines are perpendicular

So that $(-3)\cdot\left(-\frac{3P}{7}\right)+\frac{2P}{7}\cdot 1+2\cdot(-5) = 0$

$$\Rightarrow \frac{9P}{7}+\frac{2P}{7} = 10$$

$$\Rightarrow 11P = 70$$

$$\Rightarrow P = \frac{70}{11}$$

4 Ans : -

Comparing the given equation with the standard form of equation $\vec{r} = \vec{r_1}+\lambda\vec{m}$ and $\vec{r} = \vec{r_2}+\mu\vec{m}$ we, get

$\vec{r_1} = \hat{i}+2\hat{j}-4\hat{k}$

$\vec{r_2} = 3\hat{i}+3\hat{j}-5\hat{k}$

$\vec{m} = 2\hat{i}+3\hat{j}+6\hat{k}$

$\therefore |\vec{m}| = \sqrt{4+9+36} = \sqrt{49} = 7$

$\vec{r_2}-\vec{r_1} = 2\hat{i}+\hat{j}-\hat{k}$

$$\therefore \vec{m}\times(\vec{r_2}-\vec{r_1}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= (-3-6)\hat{i}+(12+2)\hat{j}+(2-6)\hat{k}$$
$$= -9\hat{i}+14\hat{j}-4\hat{k}$$

$$(\textit{shortest distance between the lines})\, d = \left|\frac{\vec{m}\times(\vec{r_2}-\vec{r_1})}{|\vec{m}|}\right|$$

$$= \frac{|-9\hat{i}+14\hat{j}-4\hat{k}|}{7}$$

$$= \frac{\sqrt{81+196+16}}{7}$$

$$= \frac{\sqrt{293}}{7} \text{ units.}$$

5 Ans : -

Let O(0,0,0) be the origin. The d.r.s of given lines are a=2,b=-3,c=4

$\therefore$ The length of the perpendicular is

$$P = \left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{2.0+(-3).0+4.0-6}{\sqrt{4+9+16}}\right|$$

$$= \frac{|-6|}{\sqrt{29}}$$

$$= \frac{6}{\sqrt{29}} \text{ units}$$

6 Ans : -

The given planes are

$3x-6y-2z=7$ and $2x+y-2z=5$

So that, the normal of the planes are

$\vec{n_1} = 3\hat{i}-6\hat{j}-2\hat{k}$ and $\vec{n_2} = 2\hat{i}+\hat{j}-2\hat{k}$

Since, angle between the planes is equal to angle between the normals

$$\therefore \cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$$

$$= \frac{6-6+4}{\sqrt{9+36+4}\sqrt{4+1+4}}$$

$$= \frac{4}{\sqrt{49}\sqrt{9}} = \frac{4}{7\cdot 3}$$

$$\cos\theta = \frac{4}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

7 Ans : -

Let the θ be the angle between planes.

The eqn. of 1st plane is x+y+2z=9

so that, d.r.s are $a_1 = 1, b_1 = 1, c_1 = 2$

The eqn. of 2nd plane is 2x-y+z=6

so that, d.r.s are $a_2 = 2, b_2 = -1, c_2 = 1$

$$\therefore \cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{2-1+2}{\sqrt{1+1+4}\sqrt{4+1+1}}$$

$$\cos\theta = \frac{3}{6} = \frac{1}{2} = \cos 60^\circ$$

$$\therefore \theta = 60^\circ$$

8 Ans : -

The given planes are

2x-4y+3z=7 and x+2y+λz=18

where $a_1 = 2, b_1 = -4, c_1 = 3$

and $a_2 = 1, b_2 = 2, c_2 = \lambda$

The given planes are perpendicular,

$$\therefore a_1a_2+b_1b_2+c_1c_2 = 0$$

$$2-8+3\lambda = 0$$

$$\lambda = \frac{6}{3} = 2$$

9 Ans : -

Let the eqn. of a plane parallel to the planes -2x+y-3z=0 is -2x+y-3z=k.

$\because$ planes -2x+y-3z=k is passes through points (1,4,-2)

$\therefore\ -2.1+4-3.(-2) = k$

$\Rightarrow\ -2+4+6 = k$

k = 8

Hence the required plane is

$-2x+y-3z=8$

10 Ans : -

Let the equation of plane be

ax+by+cz+d=0 ....(1)

The plane is passes through the point(1,1,-1) (6,4,-5) and (-4,-2,3) we, get

a+b-c+d=0 ....(2)

6a+4b-5c+d=0 ....(3)

and -4a-2b+3c+d=0 ....(4)

subtract (3)-(2) and (3)-(4) we, get

5a-5b-4c=0 ....(5)

and , 10a+6b-8c=0 ....(6)

solve eqn(5) and (6) by cross multiplication method we, get

$$\frac{a}{40+24} = \frac{b}{-40+40} = \frac{c}{30+50} = k(\text{say})$$

a=64k , b=0 , c=80k

put these values in eqn(2) we, get

64k+0-80k+d=0

d=16k

Again, put the values of a,b,c and d in eqn(1) we get

64kx+0y+80kz+16k=0

$\Rightarrow\ 64x+80z+16=0$

$\Rightarrow\ 4x+5z+1=0$

This is the required equation of plane.

5-Marks Solution

1 Ans : -

The equation of the given lines are

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

and , $$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

we know that the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda b_1$ and $\vec{r} = \vec{a_2} + \mu b_2$ is, given by

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2})(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| \text{ ....(1)}$$

comparing the given equation , we get

$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore\ \vec{a_2} - \vec{a_1} = (2\hat{i} - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$

$= \hat{i} - 3\hat{j} - 2\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= (-2 - 1)\hat{i} - (2 - 2)\hat{j} + (1 + 2)\hat{k}$

$= -3\hat{i} + 3\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-3)^2 + 3^2} = \sqrt{9 + 9} = 3\sqrt{2}$

Now ,

$$d = \left| \frac{(-3\hat{i} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} - 2\hat{k})}{3\sqrt{2}} \right|$$

$$= \left| \frac{-3 - 6}{3\sqrt{2}} \right| = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \text{ units.}$$

2 Ans : -

The equation of the given lines are

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

and , $$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

we know that the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda b_1$ and $\vec{r} = \vec{a_2} + \mu b_2$ is, given by

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2})(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

comparing the given equation , we get

$\vec{a_1} = \hat{i} + \hat{j}$

$\vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\therefore\ \vec{a_2} - \vec{a_1} = (\hat{i} - 2\hat{j} - \hat{k})$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= (-2 + 5)\hat{i} - (4 - 3)\hat{j} + (-10 + 3)\hat{k}$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{9 + 1 + 49} = \sqrt{59}$

Now ,

$$d = \left| \frac{(3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - 2\hat{j} - \hat{k})}{\sqrt{59}} \right|$$

$$= \left| \frac{3 + 2 + 7}{\sqrt{59}} \right| = \frac{12}{\sqrt{59}} = \frac{12\sqrt{59}}{59} \text{ units.}$$

3 Ans : -

The given lines are

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

and , $$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

we know that the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b}_1$ and $\vec{r} = \vec{a_2} + \mu\vec{b}_2$ is given by

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2})(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

comparing the given equation , we get

$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\therefore\ \vec{a_2} - \vec{a_1} = (3\hat{i} + 3\hat{j} + 3\hat{k})$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= (-3 - 6)\hat{i} - (1 - 4)\hat{j} + (3 + 6)\hat{k}$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{81 + 9 + 81} = \sqrt{171} = 3\sqrt{19}$

Now ,

$$d=\left|\frac{(-9\hat{i}+3\hat{j}+9\hat{k})\cdot(3\hat{i}+3\hat{j}+3\hat{k})}{3\sqrt{19}}\right|$$

$$=\left|\frac{-27+9+27}{3\sqrt{19}}\right|=\frac{9}{3\sqrt{19}}=\frac{3}{\sqrt{19}} \text{ units.}$$

4 Ans : -

The given lines are $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

These lines are comparing with standard form of equation. we, get

$x_1=-1, y_1=-1, z_1=-1$

$x_2=3, y_2=5, z_2=7$

$a_1=7, b_1=-6, c_1=1$

$a_2=1, b_2=-2, c_2=1$

$$d=\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}$$

Now ,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$=\begin{vmatrix} 3+1 & 5+1 & 7+1 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}=\begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$=4(-6+2)-6(7-1)+8(-14+6)$

$=-16-36-64$

$=-116$

Again ,

$\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}$

$=\sqrt{(-6+2)^2+(1-7)^2+(-14+6)^2}$

$=\sqrt{16+36+64}=\sqrt{116}=2\sqrt{29}$

Subsitituting all the values in eqn(1). we, get

$$\therefore d=\frac{-116}{2\sqrt{29}}=\frac{-58}{\sqrt{29}}=\frac{-58\sqrt{29}}{29}=-2\sqrt{29}$$

$$=2\sqrt{29} \text{ unit } (\because \text{distance is always non - negative})$$

5 Ans : -

The equation of the plane passing through the intersection of the planes 3x-y+2z-4=0 and x+y+z-2=0 is

$(3x-y+2z-4)+\lambda(x+y+z-2)=0 \ ,\lambda\epsilon R$ ....(1)

The plane passes through the point (2,2,1), therefore this point will satisfy the (1).

$(3.2-2+2.1-4)+\lambda(2+2+1-2)=0$

$\Rightarrow (6-2+2-4)+\lambda(3)=0$

$\Rightarrow 2+3\lambda=0$

$\therefore \lambda=-2/3$

Substituting $\lambda=-2/3$ in eqn(1) , we obtain

$(3x-y+2z-4)-\frac{2}{3}(x+y+z-2)=0$

$\Rightarrow 9x-3y+6z-12-2x-2y-2z+4=0$

$\Rightarrow 7x-5y+4z-8=0$

This is the required equation of plane.

6 Ans : -

The equation of plane passing through the intersection of the planes x+y+z=1 and 2x+3y+4z=5 is

$(x+y+z-1)+\lambda(2x+3y+4z-5)=0, \lambda\epsilon R$

$=>(2\lambda+1)x+(3\lambda+1)y+(4\lambda+1)z+(-5\lambda-1)=0$ ....(1)

The d.r.s a_1,b_1,c_1 of the plane (1) are $(2\lambda+1),(3\lambda+1),(4\lambda+1)$ respectively.

The plane of(1) is perpendicular to x-y+z=0 Its d.r.s a_2,b_2,c_2 are 1,-1and 1 respectively.

$\because$ the planes are perpendicular

$\therefore$ $a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow (2\lambda+1)\cdot 1+(3\lambda+1)\cdot(-1)+(4\lambda+1)\cdot 1=0$

$\Rightarrow 2\lambda+1-3\lambda-1+4\lambda+1=0$

$\Rightarrow 3\lambda+1=0$

$\Rightarrow \lambda=\frac{-1}{3}$

put $\lambda=\frac{-1}{3}$ in(1) we get

$$\left(\frac{-2}{3}+1\right)x+\left(\frac{-3}{3}+1\right)y+\left(\frac{-4}{3}+1\right)z+\left(\frac{5}{3}-1\right)=0$$

$$\Rightarrow \frac{1}{3}x+0.y-\frac{1}{3}z+\frac{2}{3}=0$$

$$\Rightarrow x-z+2=0$$

This is required eqn. of plane.

7 Ans : -

The eqn. of planes are

$\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7=0$ ....(1)

and $\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9=0$ ....(2)

The eqn. of plane passing through the intersection of the planes (1) and (2) is, given by

$$\left[\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})-7\right]+\lambda\left[\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9\right]=0$$

$$\Rightarrow \vec{r}\cdot\left[(2\hat{i}+2\hat{j}-3\hat{k})+\lambda(2\hat{i}+5\hat{j}+3\hat{k})\right]=9\lambda+7$$

$$\Rightarrow \vec{r}\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right]=9\lambda+7$$

.....................(3)

The plane passes through the point (2,1,3)

$\therefore$ its position vector is $\vec{r}=2\hat{i}+\hat{j}+3\hat{k}$

Subsitituting in eqn(3) we, obtain

$$(2\hat{i}+\hat{j}+3\hat{k})\cdot\left[(2+2\lambda)\hat{i}+(2+5\lambda)\hat{j}+(-3+3\lambda)\hat{k}\right]=9\lambda+7$$

$$\Rightarrow 2(2+2\lambda)+(2+5\lambda)+3(-3+3\lambda)=9\lambda+7$$

$$\Rightarrow 4+4\lambda+2+5\lambda-9+9\lambda=9\lambda+7$$

$$\Rightarrow -3+18\lambda=9\lambda+7$$

$$\Rightarrow 9\lambda=10 \Rightarrow \lambda=\frac{10}{9}$$

Substituting $\lambda=\frac{10}{9}$ in eqn (3). we, get

$$\vec{r}\cdot\left[\left(2+\frac{20}{9}\right)\hat{i}+\left(2+\frac{50}{9}\right)\hat{j}+\left(-3+\frac{30}{9}\right)\hat{k}\right]=\frac{90}{9}+7$$

$$\Rightarrow \vec{r}\cdot\left[38\hat{i}+68\hat{j}+3\hat{k}\right]=153$$

This is required equation of plane.

8 Ans : -

Given equation of plane are

$\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=5$ and $\vec{r}\cdot(3\hat{i}-3\hat{j}+5\hat{k})=3$

We know that if $\vec{n_1}$ and $\vec{n_2}$ are normal to the planes $\vec{r}\cdot\vec{n_1}=d_1$ and $\vec{r}\cdot\vec{n_2}=d_2$ then the angle between them θ, is given by

$$\cos\theta=\left|\frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\cdot|\vec{n_2}|}\right| \text{........} (1)$$

Here, $\vec{n_1}=2\hat{i}+2\hat{j}-3\hat{k}$

$\vec{n_2}=3\hat{i}-3\hat{j}+5\hat{k}$

$\vec{n_1}\cdot\vec{n_2}=6-6-15=-15$

$|\vec{n_1}|=\sqrt{4+4+9}=\sqrt{17}$

$|\vec{n_2}|=\sqrt{9+9+25}=\sqrt{43}$

$$\therefore \cos\theta=\left|\frac{-15}{\sqrt{17}\cdot\sqrt{43}}\right|=\frac{15}{\sqrt{731}}$$

$$\therefore \theta=\cos^{-1}\frac{15}{\sqrt{731}} \quad \textit{Ans...}$$

9 Ans : -

Let the equation of required plane be

ax+by+cz+d=0 ....(1)

It is perpendicular to the plane x+y-2z=6

$\therefore \quad a\cdot 1+b\cdot 1+c\cdot(-2)=0$

$\Rightarrow \quad a+b-2c=0$ .... (2)

$\because$ the plane (1) passes through the given points(1,-2,4) and (3,-4,5). then,

a-2b+4c+d=0 ....(3)

and 3a-4b+5c+d=0 ....(4)

Subtract (3) from (4) we, gets

2a-2b+c=0 ....(5)

Now from (2) and (5)

a+b-2c=0

2a-2b+c=0

Solving these equation by cross multiplication method we, get

$$\frac{a}{1-4}=\frac{b}{-4-1}=\frac{c}{-2-2}$$

$$\Rightarrow \frac{a}{-3}=\frac{b}{-5}=\frac{c}{-4}=-k(\text{say})$$

$$\Rightarrow \frac{a}{3}=\frac{b}{5}=\frac{c}{4}=k$$

$\Rightarrow a=3k$, $b=5k$, $c=4k$

Substituting these values in (3). we, get

3k-10k+16k+d=0

9k+d=0

d=-9k

Putting the value of a,b,c and d in (1) we, get

3x+5y+4z–9 = 0

Hence, the required plane.

Chapter - 12 LINEAR PROGRAMMING PROBLEMS
रैखिक प्रोग्रामन

Solve the L.P.P graphically.

आलेख द्वारा निम्न रैखिक प्रोग्रामन समस्या को हल कीजिएः

1. Maximize $z = 4x + y$ s.t. $x+y \leq 50$,

$3x+y \leq 90$

$x \geq 0, \ y \geq 0$

निम्न अवरोधो के अंतर्गत $z = 4x + y$ का अधिकतम मान ज्ञात कीजिए : $x + y \leq 50, 3x + y \leq 90$

$x \geq 0, \ y \geq 0$

2. Minimize $Z = -3x + 4y$ subject to $x + 2y \leq 8, 3x + 2y \leq 12,$

$x \geq 0, y \geq 0$

निम्न अवरोधो के अंतर्गत $Z = -3x + 4y$ का न्यूनतमीकरण कीजिए :

$x + 2y \leq 8, 3x + 2y \leq 12,$

$x \geq 0, y \geq 0$

3. Maximize $Z = 3x + 2y$ subject to

$x + 2y \leq 10, 3x + y \leq 15 \quad x \geq 0, y \geq 0$

निम्न अवरोधो के अंतर्गतः $Z = 3x + 2y$ का अधिकतम मान ज्ञात कीजिए –

$x + 2y \leq 10, 3x + y \leq 15 \quad x \geq 0, y \geq 0$

4. Find Maximum and Minimum value of $Z = 5x + 10y$ subject to $x + 2y \leq 120$,

$x + y \geq 60, x - 2y \geq 0 \ \ x, y \geq 0$

निम्न अवरोधो के अंतर्गत $Z = 5x + 10y$ का न्यूनतमीकरण तथा अधिकतमीकरण कीजिए :

$x + 2y \leq 120$,

$x + y \geq 60, x - 2y \geq 0 \ \ x, y \geq 0$

5. Maximize $Z = y - 2x$ subject to $x \leq 2, x + y \leq 3, -2x + y \leq 1 \quad x, y \geq 0$

निम्न अवरोधो के अंतर्गत $Z = y - 2x$ का अधिकतमीकरण कीजिए –

$x \leq 2, x + y \leq 3, -2x + y \leq 1 \quad x, y \geq 0$.

6. Minimize and Maximize $Z = x + 2y$ subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200$.

$x, y \geq 0$

निम्न अवरोधों के अंतर्गत $Z = x + 2y$ का न्यूनतमीकरण तथा अधिकतमीकरण कीजिए :

$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200$

$x, y \geq 0$

5 MARKS SOLUTION

1.

Sol -

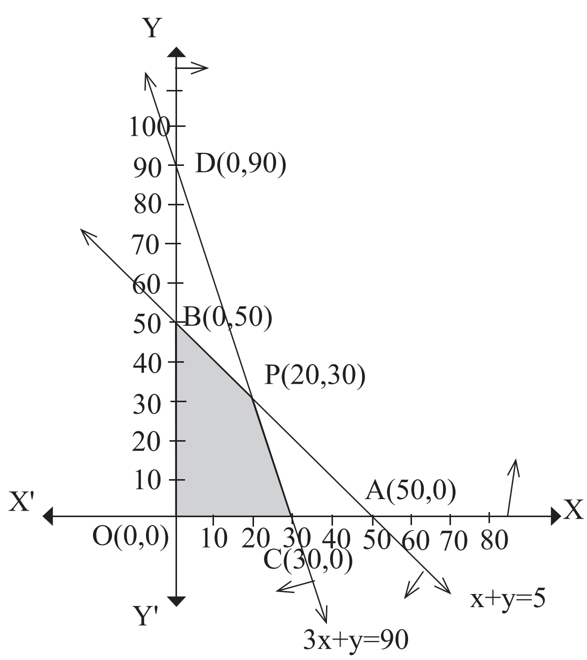
Convert all the constraints into equation, we get,

$x + y = 50, \ 3x + y = 90, x = 0, y = 0$

$\Rightarrow \frac{x}{50} + \frac{y}{50} = 1, \frac{x}{30} + \frac{y}{90} = 1$

Hence Eq^n $x + y = 50$ cuts the $x - axis$ at $A(50,0)$ and $y - axis$ at $B(0,50)$

Similarly eq^n $3x + y = 90$ cuts the $x - axis$ at $C(30,0)$ and $y - axis$ at $D(0,90)$.



Check the region at (0,0) we get

$0 + 0 \leq 50$ & $0 + 0 \leq 90$

$\therefore$ all inequation satisfies (0,0)

Corner point	objective function Z_{max}=4x+y
O(0,0)	Z=0
C(30,0)	Z=4.30+0=120 (Max^m)
P(20,30)	Z=4.20+30=110
B(0,50)	Z=0+50=50

$\therefore$ Z is maximum at C(30,0)

$\therefore Z_{max}$=120

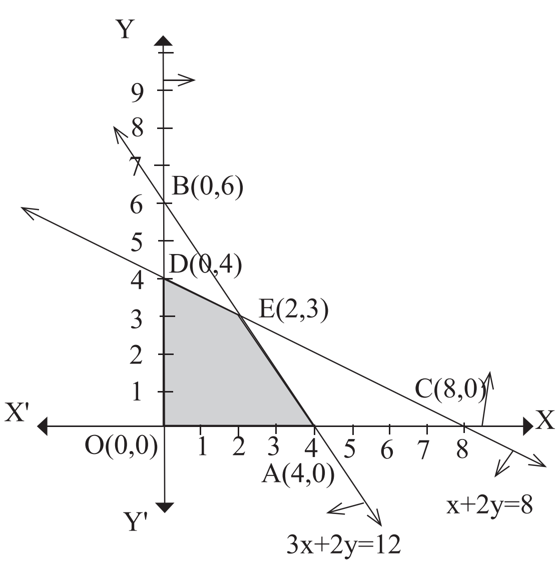
2. Convert all the constraints into eq^n-

$x + 2y = 8$, $3x + 2y = 12$, $x = 0, y = 0$

Draw the graph-

x	8	0
y	0	4

x	4	0
y	0	6



put (0,0) into equation

we get, $0 \leq 8$, $0 \leq 12$

All the euation satisfies (0,0)

The corner points of the feasible region are

O(0,0), A(4,0) , E(2,3) and D(0,4)

The values of Z at there corner points are-

Corner points	Z=-3x+4y
O(0,0)	Z=0
A(4,0)	Z=-12+0=-12 (Min^m)
E(2,3)	Z=-6+12=6
D(0,4)	Z=0+16=16

The minimum value of Z is -12 at (4,0).

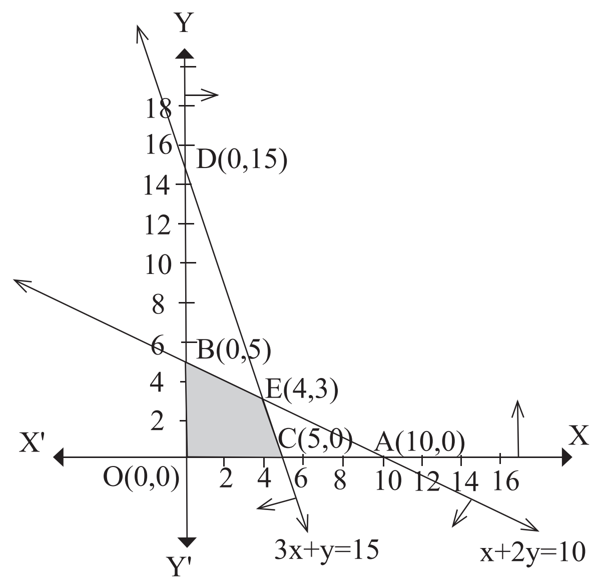
3. Convert all the constraints into eq^n.

$x + 2y = 10$, $3x + y = 15$, $x = 0, y = 0$

Draw the graph-

x	10	0
y	0	5

x	5	0
y	0	15



The corner points of the feasible region are

O(0,0), C(5,0) , E(4,3) and B(0,5)

The values of Z at these corner points are-

Corner points	Z=3x+2y
O(0,0)	Z=0+0=0
C(5,0)	Z=3.5+0=15
E(4,3)	Z=3.4+2.3=18 (Max^m)
B(0,5)	Z=0+2.5=10

$\therefore$ The maximum value of Z is 18 at the point (4,3)

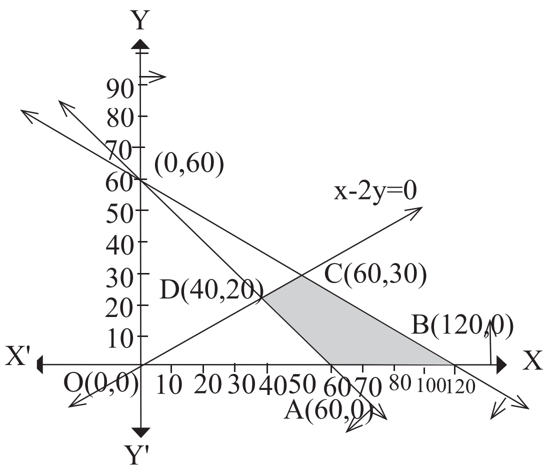
4. Convert all the constraints into eq^n-

We get,

$x + 2y = 120$, $x + y = 60$, $x - 2y = 0$,
$x = y = 0$

$$\Rightarrow \frac{x}{120} + \frac{y}{60} = 1,\ \frac{x}{60} + \frac{y}{60} = 1,$$
$x = 2y$

Hence the equation x+2y=120 cut at (120,0) and (0, 60) and x+y=60 cuts x-axis at (60, 0) and y-axis at (0,60) and x=2y passes through the origin and all equation satisfies (0.0)



The corner points of the feasible region are A(60,0) , B(120,0) , C(60,30) and D(40,20)

The value of z at these corner points are-

Corner points	Z=5x+10y
A(60,0)	Z=5.60+0=300 (Min^m)
B(120,0)	Z=5.120+0=600 (Max^m)
C(60,30)	Z=5.60+10.30=600(Max^m)
D(40,20)	Z=5.40+10.20=400

The minimum value of Z is 300 at (60,0) and the maximum value of Z is 600 at all the points on the line segment joining (120,0) and (60,30).

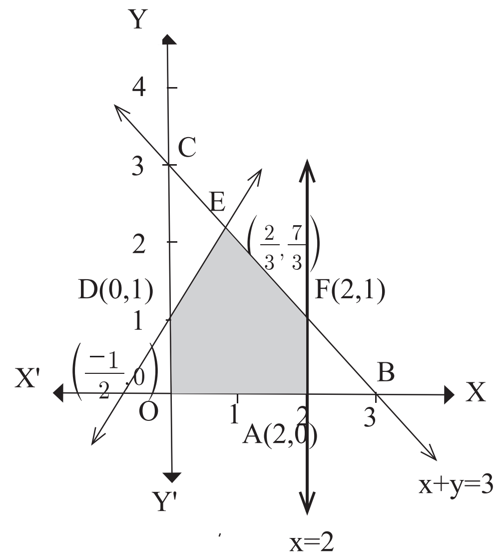
5. Sol- Convert all the constraints into eq^n-

We, get

$x = 2, x + y = 3$, $-2x + y = 1$, $x = 0, y = 0$

$$\Rightarrow \frac{x}{3} + \frac{y}{3} = 1,\ \frac{x}{\left(-\frac{1}{2}\right)} + \frac{y}{1} = 1$$

Hence, the equation x=2 cut x-axis at (2,0) and parallel to y-axis, eq^n x+y=3 cuts x-axis at (3,0) and y-axis at (0,3) and -2x+y=1 cuts x-axis at $\left(-\frac{1}{2},0\right)$ and y-axis at (0,1) and all inequation satisfies at (0,0).



The corner points of the feasible region are O(0,0), A(2,0), F(2,1), E$\left(\frac{2}{3},\frac{7}{3}\right)$ and D(0,1)

The value of Z at these corner points are-

Corner points	Z=y-2x
O(0,0)	0+0=0
A(2,0)	0-4=-4
F(2,1)	1-4=-3
E(2/3,7/3)	$\frac{7}{3} - \frac{4}{3} = 1$ (Max^m)
D(0,1)	1-0=1 (Max^m)

The maximum value of Z is 1 at all points on the line segment joining D(0,1) and E($\frac{2}{3}$ - $\frac{7}{3}$)

6. Convert all the constraints into eq^n-

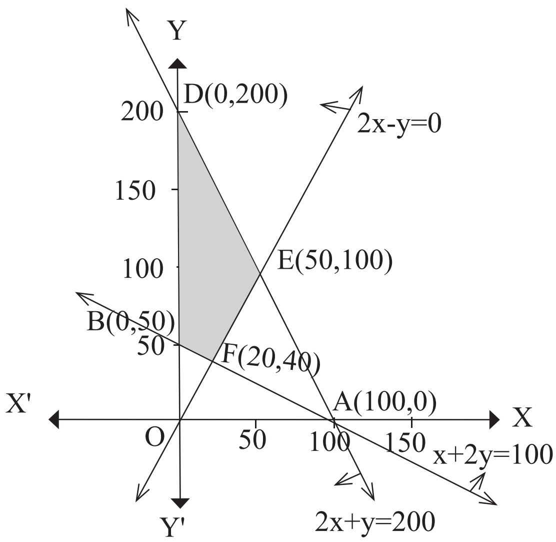
We get,

$x + 2y = 100$, $2x - y = 0$, $2x + y = 200$,
$x = y = 0$

$$\Rightarrow \frac{x}{100} + \frac{y}{50} = 1,\ 2x = y \quad \frac{x}{100} + \frac{y}{200} = 1$$

Equation x+2y=100 cuts x-axis at points A(100,0) and y axis at B(0,50).

Equation 2x=y passes through origin and equation 2x+y=200 cuts x-axis at point C(100,0) and y-axis at D(0,200) and all the equation satisfies (0,0).



The Corner points of the feasible region are B(0,50), E(50,100), F(20,40) and D(0,200).

The value of Z at these corner points are -

Corner points	Z=x+2y
B(0,50)	Z= $0+2.50=100$ (Min^{m})
F(20,40)	Z=20+80=100 (Min^{m})
E(50,100)	Z=50+200=250
D(0,200)	Z=0+400=400 (Max^{m})

The Maximum value of Z is 400 at (0,200) and the minimum value of Z is 100 at all points on the line segment joining the points B(0,50) and F(20,40).

Chapter : 13 PROBABILITY प्रायिकता

MCQ

1 An event in the probability that will never be happened is called as.

एक घटना जो कभी नहीं होगी उसे क्या कहा जाता हैं ?

a) Unsure event अनिश्चित घटना b) Sure event निश्चित घटना

c) Possible event सम्भव घटना d) Impossible event असम्भव घटना

2. What will be the probability of getting odd numbers if a dice is thrown ?

एक पासे को फेंकने पर विषम संख्या आने की प्रायिकता क्या होगी ?

a) $\frac{1}{2}$ b) 2

c) $\frac{1}{3}$ d) $\frac{5}{2}$

3. What will be the probability of losing a game if the winning probability is 0.3 ?

यदि एक खेल के जीतने की प्रायिकता 0.3 है तो इसे हारने की प्रायिकता क्या हागी ?

a) 0.5 b) 0.6

c) 0.7 d) 0.8

4. A card is drawn from a pack of 52 cards.What is the probability of getting a king of a black suit ?

52 ताश के पत्तो की एक गड्डी से एक काला राजा मिलने की प्रायिकता क्या है?

a) $\frac{1}{52}$ b) $\frac{1}{26}$

c) $\frac{3}{26}$ d) $\frac{7}{52}$

5. A card is drawn from a pack of 52 cards.What is the probability of getting a queen card ?

52 ताश के एक गड्डी से एक पत्ता निकाला गया। उसके रानी होने की प्रायिकता क्या है?

a) $\frac{1}{26}$ b) $\frac{1}{52}$

c) $\frac{3}{13}$ d) $\frac{1}{13}$

6. Which of the following can be the probability of an event ?

निम्न में से कौन सी संख्या किसी घटना की प्रायिकता हो सकती हैं।

a) -1.3 b) 04

c) $\frac{3}{8}$ d) $\frac{10}{7}$

7. If P(A)=$\frac{1}{2}$, P(B)=0 then P $\left(\frac{A}{B}\right)$ is

यदि P(A)=$\frac{1}{2}$, P(B)=0 तब P $\left(\frac{A}{B}\right)$ है।

a) 0 b) $\frac{1}{2}$,

c) Undefined परिभाषित नहीं d) 1

8. If A and B be two events such that

$P\left(\frac{A}{B}\right)=P\left(\frac{B}{A}\right)\neq 0$, then

यदि A और B दो घटनाएँ इस प्रकार है कि

$P\left(\frac{A}{B}\right)=P\left(\frac{B}{A}\right)\neq 0$, तब

a) $A\subset B$ b) A=B

c) $A\cap B\neq\phi$ d) P(A)=P(B)

9. If E and F are events such that P(E)=0.6, P(F)=0.3 and $P(E\cap F)=0.2$. Find $P\left(\frac{E}{F}\right)$

यदि E और F इस प्रकार की घटनाएँ है कि P(E)=0.6, P(F)=0.3 और $P(E\cap F)=0.2$. हैं तो $P\left(\frac{E}{F}\right)$. होगा।

a) $\frac{2}{3}$ b) $\frac{3}{2}$

c) $\frac{1}{6}$ d) $\frac{1}{2}$

10. $P(A\cap B)$ is equal to.

$P(A\cap B)$ के बराबर होगा।

a) $P(A).P\left(\frac{B}{A}\right)$ b) $P(B).P\left(\frac{A}{B}\right)$

c) Both (a) and (b)
(a) और (b) दोनों

d) None of these
इनमें से कोई नहीं

11. If E and F are independent events , then
यदि E और F स्वतंत्र घटना हो, तो

a) $P(E \cap F) = \frac{P(E)}{P(F)}$

b) $P(E \cap F) = P(E) + P(F))$

c) $P(E \cap F) = P(E).P(F)$

d) None of these
इनमें से कोई नहीं

12. The probability of obtaining an even prime number on each die when a pair of dice rolled.
दो पासे के एक बार फेकने पर प्रत्येक में सम अभाज्य संख्या आने की प्रायिकता क्या होगी?

a) 0 b) $\frac{1}{3}$

c) $\frac{1}{12}$ d) $\frac{1}{36}$

13. A die is thrown once , then the probability of getting a number greater than 3 is
एक पासे को एक बार फेंकने पर 3 से बड़ी संख्या आने की प्रायिकता होगी।

a) $\frac{1}{2}$ b) $\frac{2}{3}$

c) 3 d) 0

14. An urn countaining 6 balls of which two are red and four are black .Two balls are drawn at random, probability that they are of the different colour is?
एक थैले में 6 गेंदे है जिसमें 2 गेंदे लाल और 4 गेंदे काली है तो दो गेंदे निकालने पर दोनों भिन्न रंग के होने की प्रायिकता क्या होगी ?

a) $\frac{2}{5}$ b) $\frac{1}{15}$

c) $\frac{8}{15}$ d) $\frac{4}{15}$

2 MARKS QUESTION

1. Find $P\left(\frac{A}{B}\right)$. If P(B)=0.5 and $P(A \cap B) = 0.32$
$P\left(\frac{A}{B}\right)$ ज्ञात कीजिए, यदि P(B)=0.5 और $P(A \cap B) = 0.32$

2. If P(A)=$\frac{6}{11}$, P(B)=$\frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, then find $P(A \cap B)$.
यदि P(A)=$\frac{6}{11}$,P(B)=$\frac{5}{11}$ और $P(A \cup B) = \frac{7}{11}$ हो तो $P(A \cap B)$. का मान ज्ञात करे ।

3. Given two independent events A and B Such that P(A)=0.3 P(B)=0.6 .Find P(A and B).
A और B स्वतंत्र घटनाएँ दी गई है जहाँ P(A)=0.3, P(B)=0.6 तो P(A और B) का मान ज्ञात करे ।

4. If P(A)=0.8 , P(B)=0.5 and $P\left(\frac{B}{A}\right) = 0.4$, find the following :-
यदि P(A)=0.8 , P(B)=0.5 और $P\left(\frac{B}{A}\right) = 0.4$, तो निम्नलिखित ज्ञात करें :–
(i) $P(A \cap B)$ (ii) P(A/B)
(iii) $P(A \cup B)$

5. Two cards are drawn at random and without replacement from a pack of 52 playing card . Find the probability that both the cards are black.
52 पत्तो की एक गड्डी में से यादृच्छया बिना प्रतिस्थापित किए दो पत्ते निकाले गए । दोनो पत्तों के काले रंग का होने की प्रायिकता ज्ञात कीजिए।

6. A die marked 1,2,3 in red and 4,5,6 in green is tossed . Let A be the event 'the number is even' and B be the event 'the number is red' ,Are A and B independent ?
एक पासे पर 1, 2,3 लाल रंग से और 4, 5, 6 हरे रंग से लिखे गए है। इस पासे को उछाला गया । मान ले A घटना ''संख्या सम'' है और B घटना ''संख्या लाल रंग से लिखी गई है'' को निरूपित करते हैं। क्या A और B स्वतंत्र है?

7. If E_1 and E_2 are two independent events such that $P(E_1) = 0.35$ and $P(E_1 \cup E_2) = 0.60$ find $P(E_2)$.
यदि E_1 और E_2 दो स्वतंत्र घटना इस प्रकार है कि $P(E_1) = 0.35$ और $P(E_1 \cup E_2) = 0.60$ तो $P(E_2)$ का मान ज्ञात करें।

3 MARKS QUESTION

1. Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \frac{5}{13}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$

$P(A \cup B)$ ज्ञात कीजिए, यदि

$2P(A) = P(B) = \frac{5}{13}$ और $P\left(\frac{A}{B}\right) = \frac{2}{5}$.

2. Detrmine $P\left(\frac{E}{F}\right)$ if a coin is tossed three times

Where E: head on third toss & F : head on first two tosses.

$P\left(\frac{E}{F}\right)$ ज्ञात कीजिए । यदि एक सिक्के को तीन बार उछाला गया है, जहाँ E= तीसरी उछाल पर चित F=पहली दोनो उछालो पर चित।

3. A family has two children .What is the probability that both the childrens are boys given that at least one of them is a boy?

एक परिवार में दो बच्चे हैं। यदि यह ज्ञात हो कि बच्चों में से कम से कम एक बच्चा लड़का है तो दोनो बच्चों के लड़का होने की क्या प्रायिकता है ?

4. A die is thrown twice and the sum of the numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once ?

एक पासे को दो बार उछाला गया और प्रकट हुई संख्याओ का योग 6 पाया गया। संख्या 4 के न्यूनतम एक बार प्रकट होने की सप्रतिबंध प्रायिकता ज्ञात कीजिए।

5. A fair coin and an unbiased die are tossed .Let A be the event 'head appears on the coin' and B be the event '3 on the die'.Check whether A and B are independent event or not?

एक न्याय्य सिक्का और एक अभिनत पासे को उछाला गया। मान ले A घटना 'सिक्के पर चित प्रकट होता है' और B घटना पासे पर संख्या '3 प्रकट होती है' को निरूपित करते है। निरीक्षण कीजिए कि घटनाएँ A और B स्वतंत्र हैं या नहीं?

6. Let A and B be independent events with

P (A) = 0.3 and P (B) = 0.4 then find

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P\left(\frac{B}{A}\right)$

मान ले A और B स्वतंत्र घटनाएँ है तथा P(A)=0.3 और P(B)=0.4 तब

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P\left(\frac{B}{A}\right)$

ज्ञात कीजिए।

7. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls .Find the probability that both balls are red.

दो गेंद एक बॉक्स से बिना प्रतिस्थापित किए निकाली जाती है। बॉक्स में 10 काली और 8 लाल गेंदे है तो दोनो गेंदे लाल हाने की प्रायिकता ज्ञात कीजिए।

5 MARKS QUESTION

1. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bag is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

एक थैले में 4 लाल और 4 काली गेंदे है और एक अन्य थैले में 2 लाल और 6 काली गेंदें है। दोनों थैलों में से एक को यादृच्छया चुना जाता है और उसमें एक गेंद निकाली जाती है जो लाल है। इस बात की क्या प्रायिकता है कि गेंद पहले थैले से निकाली गई है ?

2. An insurance company insured 2000 scooter driver, 4000 car drivers and 6000 truck drivers. The probability of accident are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident .What is the probability that he is a scooter driver.

एक बीमा कंपनी 2000 स्कूटर चालकों, 4000 कार चालकों और 6000 ट्रक चालकों का बीमा करती है। दुर्घटनाओं की प्रायिकताएँ क्रमशः 0.01,0.03 और 0.15 है। बीमाकृत व्यक्तियों में से एक दुर्घटनाग्रस्त हो जाता है। उस व्यक्ति के स्कूटर चालक होने की प्रायिकता क्या है?

3. From a lot of 30 bulbs which include 6 defectives . A sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

30 बल्बो के एक ढेर में से 6 बल्ब खराब है, 4 बल्बों का एक नमूना (प्रतिदर्श) यादृच्छया बिना प्रतिस्थापना के निकाला जाता है। खराब बल्बों की संख्या का प्रायिकता बंटन ज्ञात कीजिए ।

4. Find the probability distribution of the number of successesive two tosses of a die, where a success is defined as

(i) Number greater than 4

(ii) Six appears on at least one die

एक पासे दो बार उछालने पर सफलता की संख्या का प्रायिकता बंटन ज्ञात कीजिए जहाँ–

(i) '4 से बडी संख्या' को एक सफलता माना गया है।

(ii) 'पासे पर संख्या 6 प्रकट होना' को एक सफलता माना गया है।

5. Find the mean number of heads in three tosses of a fair coin.

एक न्याय्य सिक्के की तीन उछालों पर प्राप्त चितों संख्या का माध्य ज्ञात कीजिए।

6. A bag consists of 10 balls each marked with one of the digits 0 to 9 . If four balls are drawn successive with replacement from the bag, what is the probability that none is marked with the digit 0 ?

एक थैले में 10 गेंदे है जिनमें से प्रत्येक पर 0 से 9 तक के अंको में से एक अंक लिखा है। यदि थैले में 4 गेंद उत्तरोत्तर पुनः वापस रखते हुए निकाली जाती तो इसकी क्या प्रायिकता है कि उनमें से किसी भी गेंद पर अंक 0 न लिखा हो?

7. A card from a pack of 52 cards is lost .From the remaining cards of the pack , two cards are drawn and are found to be both diamonds .Find the probability of the lost card being a diamond

52 ताशों की गड्डी से एक पत्ता खो जाता है। शेष पत्तो से दो पत्ते निकाले जाते है जो ईंट के पत्ते है। खो गए पत्ते की ईंट होने की प्रायिकता क्या है ?

8. Find the probability of throwing atmost 2 sixes in 6 throws of a single dice.

एक पासे को छः बार उछालने पर अधिकतम 2 बार छः आने की प्रायिकता ज्ञात कीजिए ।

(MCQ ANSWER)

Ans- 1)-d ,2)-a , 3)-c, 4)-b, 5)-d, 6)-c 7)-c, 8)-d, 9)-a, 10)-c, 11)-c, 12)-d, 13)-a, 14)-c

(2 MARKS) SOLUTION

1. Sol - Given $P(B) = 0.5$

and $P(A \cap B) = 0.32$

$$\therefore P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25} \text{ Ans}$$

2 Sol - Given that

$$P(A) = \frac{6}{11}, P(B) = \frac{5}{11}$$

$$and\ P(A \cup B) = \frac{7}{11}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11} \text{ Ans}$$

3 Sol - Given that

$P(A) = 0.3$

, $P(B) = 0.6$

$\because$ A and B are independent events then,

$$P(A \text{ and } B) \text{ or } P(A \cap B) = P(A).P(B)$$

$$= (0.3).(0.6)$$

$$= 0.18 \text{ Ans}$$

4. Given that $P(A) = 0.8, P(B) = 0.5, P\left(\frac{B}{A}\right) = 0.4$

(i) $P\left(\frac{B}{A}\right) = 0.4$

$$\Rightarrow \frac{P(A \cap B)}{P(A)} = 0.4$$

$$\Rightarrow \frac{P(A \cap B)}{0.8} = 0.4$$

$$\therefore P(A \cap B) = 0.4 \times 0.8$$

$$= 0.32$$

(ii) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{0.32}{0.5} = 0.64$$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= 0.8 + 0.5 - 0.32$$

$$= 1.3 - 0.32 = 0.98$$

5. Sol - There are 26 black cards in a pack of 52 cards .

let $P(A)$ be the probability of getting a black cards in first draw

$$\therefore P(A) = \frac{26}{52} = \frac{1}{2}$$

Let $P(B)$ be the probability of getting a black card in the second draw

(the card is not replaced)

$$\therefore P(B) = \frac{25}{51}$$

Thus, the probability of getting both the cards black $= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

6. Sol - When a die is thrown, the sample space is

$S = \{1,2,3,4,5,6\}$, $n(S) = 6$

let A: the number is even = $\{2,4,6\}$, $n(A) = 3$

$\Rightarrow P(A) = \frac{3}{6} = \frac{1}{2}$

B: the number is red = $\{1,2,3\}$, $n(B) = 3$

$\Rightarrow P(B) = \frac{3}{6} = \frac{1}{2}$

$A \cap B = \{2\}$

$\Rightarrow P(A \cap B) = \frac{1}{6}$

now, $P(A \cap B) = \frac{1}{6}$

$\Rightarrow P(A).P(B) = \frac{1}{6}$

$\Rightarrow \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{6}$

$\Rightarrow \frac{1}{4} \neq \frac{1}{6}$

$P(A).P(B) \neq P(A \cap B)$

therefore A and B are not independent event.

7. Sol - Let $P(E_2) = x$; E_1 and E_2 being independent event

$\therefore P(E_1 \cap E_2) = P(E_1).P(E_2) = 0.35x$

$\Rightarrow P(E_1) + P(E_2) - P(E_1 \cup E_2) = 0.35x$

$\Rightarrow \quad 0.35 + x - 0.60 = 0.35x$

$\Rightarrow \quad 0.65x = 0.25$

$\Rightarrow \quad x = \frac{25}{65} = \frac{5}{13}$ Ans

3 MARKS SOLUTION

1. Sol - Given $2P(A) = P(B) = \frac{5}{13}$

so, $P(A) = \frac{5}{26}$

& $P(B) = \frac{5}{13}$

Also, $P(A/B) = \frac{2}{5}$

We know $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

$\therefore \quad P(A \cap B) = P\left(\frac{A}{B}\right).P(B)$

$= \frac{2}{5} \cdot \frac{5}{13}$

$= \frac{2}{13}$

Also, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{3}{13}$

$= \frac{5+10-4}{26} = \frac{11}{26}$ Ans

2. The sample space of the given experiment will be

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$E = \{HHH, HTH, THH, TTH\}$

$F = \{HHH, HHT\}$

$\therefore E \cap F = \{HHH\}$

so that, $n(S) = 8$, $n(E) = 4$,

$n(F) = 2$, $n(E \cap F) = 1$

$P(E) = \frac{4}{8} = \frac{1}{2}$

$P(F) = \frac{2}{8} = \frac{1}{4}$

and $P(E \cap F) = \frac{1}{8}$

$\therefore P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{8}}{\frac{1}{4}} = \frac{1}{8} \times \frac{4}{1} = \frac{1}{2}$

3. Sol - Let b stands for boy and g stands for girl.

The sample space of the experiment is

$S = \{(b,b),(g,b),(b,g),(g,g)\}$, $n(S) = 4$

Let E be the event that both children are boys & F is the event that atleast one of the child is a boy,

Then, $E = \{(b,b)\} \Rightarrow n(E) = 1$

& $F = \{(b,b),(g,b),(b,g)\} \Rightarrow n(F) = 3$

$(E \cap F) = \{(b,b)\} \Rightarrow n(E \cap F) = 1$

$\therefore P(E) = \frac{1}{4}$, $P(F) = \frac{3}{4}$, $P(E \cap F) = \frac{1}{4}$

$\therefore P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

4.

Sol -

The sample space of the given experiment is

$S = \{(1,1),(1,2),(1,3)........,(6,1),(6,2),(6,3)$
$(6,4),(6,5),(6,6)\}$

$\therefore n(S) = 36$

Let E be the event that 'number 4 appears at least once 'and F be the event that' the sum of the numbers appearing is 6'.

Then ,

$E = \{(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$
$,(1,4),(2,4),(3,4),(5,4),(6,4)\}$

$\therefore n(E) = 11$

& $F = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$

$\therefore n(F) = 5$

$\because (E \cap F) = \{(2,4),(4,2)\} \Rightarrow n(E \cap F) = 2$

Therefore $P(E) = \frac{11}{36}, P(F) = \frac{5}{36},$

$P(E \cap F) = \frac{2}{36}$

$\therefore$ Hence , the required probability

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$$

5.

Sol - If a fair coin and an unbiased die are tossed,

then $S = \begin{Bmatrix} (H.1),(H.2),(H.3),(H.4),(H.5),(H.6) \\ (T,1),(T,2),(T,3),(T,4),(T,5),(T,6) \end{Bmatrix}$

A = Head appear on the coin

$= \{(H.1),(H.2),(H.3),(H.4),(H.5),(H.6)\}$

$\therefore \quad P(A) = \frac{6}{12} = \frac{1}{2}$

B = 3 on the die

$= \{(H,3),(T,3)\}$

$\therefore P(B) = \frac{2}{12} = \frac{1}{6}$

$\therefore A \cap B = \{(H,3)\}$

$P(A \cap B) = \frac{1}{12}$

Now,

$P(A).(B) = \frac{1}{2}.\frac{1}{6} = \frac{1}{12} = P(A \cap B)$

$\because$ A and B are independent events.

6.

Sol - It is given that

$P(A) = 0.3$ and $P(B) = 0.4$

(i) $P(A \cap B) = P(A).P(B)$ [$\because$ A and B are independent events]

$= 0.3 \times 0.4 = 0.12$ Ans...

(ii) $P(A \cup B) = P(A) + P(B) - n(A \cap B)$

$= 0.3 + 0.4 - 0.12$

$= 0.7 - 0.12$

$= 0.58$ Ans...

(iii) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{0.12}{0.3} = 0.4$ Ans...

7.

Sol -

Total number of balls = 18

Number of red balls = 8

Number of black balls = 10

$\therefore$ Probability of getting a red ball in the first draw $= \frac{8}{18} = \frac{4}{9}$

$\therefore$ probability of getting a red ball in the second draw $= \frac{8}{18} = \frac{4}{9}$

$\therefore$ Probability of getting both balls red $= \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$

5 MARKS SOLUTION

1. Let E_1 and E_2 be the events of selecting first bag and second bag respectively

$\therefore P(E_1) = P(E_2) = \frac{1}{2}$

let A be the event of getting a red ball

$\therefore P\left(\frac{A}{E_1}\right) = P$(drawing a red ball from first bag)

$= \frac{4}{8} = \frac{1}{2}$

$\therefore P\left(\frac{A}{E_2}\right) = P$(drawing a red ball from second bag)

$= \frac{2}{8} = \frac{1}{4}$

The probaility of drawing a ball from the first bag , given that it is red, is given by $P\left(\frac{E_1}{A}\right)$

[By Baye's theorem]

$$\therefore P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{8}} = \frac{\frac{1}{4}}{\frac{3}{8}}$$

$$= \frac{1}{4}\times\frac{8}{3} = \frac{2}{3} \text{ Ans...}$$

2.

Sol -

Let E_1, E_2 and E_3 be the respective events that the drivers are a scooter driver, a car driver and a truck driver.

Let A be the event that the person meets with an accident

Total no of drivers = 2000 + 4000 + 6000

= 12000

$\therefore P(E_1) = \frac{2000}{12000} = \frac{1}{6}$

$P(E_2) = \frac{4000}{12000} = \frac{1}{3}$

$P(E_3) = \frac{6000}{12000} = \frac{1}{2}$

$P\left(\frac{A}{E_1}\right)$ = P(scooter driver meet with an accident)

$= 0.01 = \frac{1}{100}$

$P\left(\frac{A}{E_2}\right)$ = P(car driver meet with an accident)

$= 0.03 = \frac{3}{100}$

$P\left(\frac{A}{E_3}\right)$ = P(truck driver meet with an accident)

$= 0.15 = \frac{15}{100}$

The probability that the driver is a scooter driver is

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6}\cdot\frac{1}{100}}{\frac{1}{6}\cdot\frac{1}{100}+\frac{1}{3}\cdot\frac{3}{100}+\frac{1}{2}\cdot\frac{15}{100}}$$

$$= \frac{\frac{1}{6}}{\frac{1}{6}+1+\frac{1}{2}} = \frac{\frac{1}{6}}{\frac{1+6+45}{6}}$$

$$= \frac{1}{6}\times\frac{6}{52} = \frac{1}{52}$$

3. Sol -

It is given that out of 30 bulbs 6 are defectives.

P(defective bulb) $= \frac{1}{5}$

no of non defective bulbs = 30 - 6 = 24

P(non - defective bulb) $= \frac{4}{5}$

4 bulbs are drawn from the lot with replacement.

Let x be the random variable that denotes the number of defective bulbs in the selected bulbs.

$\therefore P(x=0)$ = P(4 - non defective and 0 defective)

$= {}^4C_0.\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$

$P(x=1)$ = P(3 non defective and 1 defective)

$= {}^4C_1.\frac{1}{5}.\left(\frac{4}{5}\right)^3 = \frac{256}{625}$

$P(x=2)$ = P(2 non defective and 2 defective)

$= {}^4C_2.\left(\frac{1}{5}\right)^2.\left(\frac{4}{5}\right)^2 = \frac{96}{625}$

$P(x=3)$ = P(1 non defective and 3 defective)a

$= {}^4C_3.\left(\frac{1}{5}\right)^3.\frac{4}{5} = \frac{16}{625}$

$P(x=4)$ = P(0 non defective and 4 defective)

$= {}^4C_4.\left(\frac{1}{5}\right)^4.\left(\frac{4}{5}\right)^0 = \frac{1}{625}$

hence, the required probability distribution is as follows –

X	0	1	2	3	4
P(x)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

4.

Sol -

When a die is tossed two time, we obtain $(6\times6) = 36$ number of observation,

Let x be the random variable, which represents the number of successes.

(i) Here, success refers to the number greater than 4.

$P(x=0)$ = P(number less than or equal to 4 on both tosses)

$= \frac{4}{6}\cdot\frac{4}{6} = \frac{4}{9}$

$P(x=1) = P\left(\begin{matrix}\text{number less than or equal to 4 on first toss}\\ \text{and greater than 4 on second toss}\end{matrix}\right)$

$+ P\left(\begin{matrix}\text{number greater than 4 on first toss and}\\ \text{less than or equal to 4 on second toss}\end{matrix}\right)$

$= \frac{4}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{4}{6} = \frac{4}{9}$

$P(x = 2) = P$ (number greater than 4 on both the tosses)

$= \frac{2}{6} \times \frac{2}{6} = \frac{1}{9}$

then the probability distribution is as follows -

x	0	1	2
P(x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success mean six appears on at least one die

$P(Y = 0) = P$ (six does not appear on any of the dice)

$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

$P(Y = 1) = P$(six appears on at least one of the dice)

$= \frac{11}{36}$

The required probability distribution is as follows

Y	0	1
P(Y)	$\frac{25}{36}$	$\frac{11}{36}$

5.

Sol -

Let x denote the success of getting head .

$\therefore$ samle space is

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

It can be seen that x can take the value of 0, 1, 2 or 3.

$\therefore P(X = 0) = P(TTT) = P(T) . P(T) . P(T)$

$= \frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} = \frac{1}{8}$

$\therefore P(X = 1) = P(HTT) + P(THT) + P(TTH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

$P(X = 2) = P(HHT) + P(HTH) + P(THH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

$P(X = 3) = P(HHH)$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

The required probability distribution is as follows -

X	0	1	2	3
P(X)	1/8	3/8	3/8	1/8

Means of X, $E(X) = M = \sum X_i P(X_i)$

$= 0.\frac{1}{8} + 1.\frac{3}{8} + 2.\frac{3}{8} + 3.\frac{1}{8}$

$= \frac{12}{8} = \frac{3}{2}$

6. Sol -

Let x denotes the number of balls marked with the digit 0 among the 4 balls drawn.

x has a binomial distribution with n = 4

and $P = \frac{1}{10}$

$\therefore q = 1 - \frac{1}{10} = \frac{9}{10}$

$\therefore P(X = x) = {}^nC_x q^{n-x} P^x, \quad x = 1, 2, 3.....n$

$\therefore P(\text{none marked with } 0) = P(X = 0)$

$= {}^4C_0 \left(\frac{9}{10}\right)^4 . \left(\frac{1}{10}\right)^0$

$= 1 . \left(\frac{9}{10}\right)^4 = \left(\frac{9}{10}\right)^4$

7. Sol -

Let E_1 and E_2 be the respective events of choosing a diamond card and a card which is not diamond.

Let A denote the lost card .

out of 52 cards , 13 cards are diamond and 39 cards are not diamond.

$\therefore P(E_1) = \frac{13}{52} = \frac{1}{4}, P(E_2) = \frac{39}{52} = \frac{3}{4}$

When one diamond card is lost -

$P\left(\frac{A}{E_1}\right) = \frac{{}^{12}C_2}{{}^{51}C_2} = \frac{12!}{2!10!} \times \frac{2!49!}{51!}$

$= \frac{11 \times 12}{50 \times 51} = \frac{22}{425}$

When one card is lost which is not diamond is

$P\left(\frac{A}{E_2}\right) = \frac{{}^{13}C_2}{{}^{51}C_2} = \frac{13!}{2!11!} \times \frac{2!49!}{51!}$

$= \frac{13 \times \cancel{12}^{2}}{{}_{17}\cancel{51} \times \cancel{50}_{25}} = \frac{26}{425}$

The probability of getting two cards,
when one card is lost which is not diamond is

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1).\,P\left(\frac{A}{E_1}\right)}{P(E_1).\,P\left(\frac{A}{E_1}\right) + P(E_2).\,P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{4}\cdot\frac{22}{425}}{\frac{1}{4}\cdot\frac{22}{425} + \frac{3}{4}\cdot\frac{26}{425}}$$

$$= \frac{22}{22+78} = \frac{22}{100} = \frac{11}{50} \text{ Ans...}$$

8.

Sol -

Let repeated tossing of the die are
Bernoulli trils .

Let X represents the number of times
of getting sixes in 6 throws of the dice.

Probability of getting six in a single
throw of dice -

$P = \frac{1}{6}$

$\therefore q = 1 - \frac{1}{6} = \frac{5}{6}$

clearly , x has a binomial distribution
with n = 6

$P(X = x) = {}^nC_x q^{n-x} p^x = {}^6C_x \left(\frac{5}{6}\right)^{6-x}\left(\frac{1}{6}\right)^x$

$P(\text{at most 2 sixes}) = P(X \le 2)$

$$= P(X = 0) + P(X = 1) + P(X = 2)$$

$$= {}^6C_0\left(\frac{5}{6}\right)^6.\left(\frac{1}{6}\right)^0 + {}^6C_1\left(\frac{5}{6}\right)^5.\frac{1}{6} + {}^6C_2\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)^2$$

$$= \left(\frac{5}{6}\right)^6 + 6.\left(\frac{5}{6}\right)^5.\frac{1}{6} + 15\left(\frac{5}{6}\right)^4.\frac{1}{36}$$

$$= \left(\frac{5}{6}\right)^4\left[\frac{25}{36} + \frac{5}{6} + \frac{5}{12}\right]$$

$$= \left(\frac{5}{6}\right)^4\left[\frac{25+30+15}{36}\right]$$

$$= \frac{70}{36}.\left(\frac{5}{6}\right)^4$$

$$= \frac{35}{18}.\left(\frac{5}{6}\right)^4 \text{ Ans....}$$

झारखंड अधिविद्य परिषद्
ANNUAL INTERMEDIATE EXAMINATION 2023

गणित

हल प्रश्न-पत्र / SOLVED PAPER

बहुविकल्पीय प्रश्नोत्तर

INSTRUCTIONS / निर्देश :

1. Carefully fill up the necessary particulars on the OMR Answer Sheet.
   सावधानी पूर्वक सभी विवरण OMR उत्तर पर पत्रक पर भरें।
2. Put your full signature on the OMR Answer Sheet in the space provided.
   आप अपना पूरा हस्ताक्षर OMR उत्तर पत्रक पर दी गई जगह पर करें।
3. There are 40 Multiple Choice Questions in this Part.
   इस भाग में कुल 40 बहु–विकल्पीय प्रश्न हैं।
4. All questions are compulsory. Each question carries 1 mark.
   सभी प्रश्नों के उत्तर देना अनिवार्य है। प्रत्येक प्रश्न की अधिमानता 1 अंक निर्धारित है।
5. There is no negative marking for any wrong answer.
   गलत उत्तर के लिए कोई अंक नहीं काटा जायेगा।
6. Use the page given at the end of the question booklet for Rough work. Do not do any Rough Work on the OMR Answer Sheet.
   रफ़ कार्य हेतु प्रश्न पुस्तिका के अंत में दिये गये पृष्ठ का ही प्रयोग कीजिए। OMR उत्तर पत्रक पर कोई रफ़ कार्य न करें।
7. Read all the instructions provided on page 2 of the OMR Answer Sheet carefully and do accordingly.
   OMR उत्तर पत्रक के पृष्ठ 2 पर प्रदत सभी निर्देशों को ध्यानपूर्वक पढ़ें तथा उसके अनुसार कार्य करें।
8. Four options are given for each question. You have to darken duly the most suitable answer on your OMR Answer Sheet. Use only Blue or Black Ball-Point Pen. The use of Pencil is not allowed.
   प्रत्येक प्रश्न में चार विकल्प दिये गये हैं। इनमें से सबसे उपयुक्त उत्तर को आप अपने OMR उत्तर पत्रक पर ठीक–ठीक गहरा काला करें। केवल नीला या काला बॉल–प्वाइंट कलम का ही प्रयोग करें। पेंसिल का प्रयोग वर्जित है।
9. Adhere to the instructions provided in the OMR Answer Sheet very carefully otherwise your OMR Answer Sheet will be treated as invalid and it will not be evaluated.
   OMR उत्तर पत्रक पर दिये गये निर्देशों का ध्यानपूर्वक पालन कीजिए अन्यथा आपका OMR उत्तर पत्रक अमान्य होगा और उसका मूल्यांकन नहीं किया जायेगा।

बहुविकल्पीय प्रश्न (MCQ)

1. What type of relation is R= {(a, b), (b, c), (b, a) , (c, b)} on the set A= {a, b, c} ?

(1) Reflexive (2) Symmetric
(3) Transitive (4) None of these

संबंध R = {(a, b), (b, c), (b, a) , (c, b)} समुच्चय A={a, b, c} पर कैसा संबंध है?

(1) स्वतुल्य (2) सममित
(3) संक्रामक (3) इनमें से कोई नहीं

2. If $f: R \rightarrow R$ be such that $f(x) = 5x + 4$, then $f^{-1}(x) =$

(1) $\frac{x-4}{5}$ (2) $\frac{4-x}{5}$
(3) $\frac{x-5}{4}$ (4) none of these

यदि $f: R \rightarrow R$ जहाँ $f(x) = 5x + 4$, तो $f^{-1}(x) =$

(1) $\frac{x-4}{5}$ (2) $\frac{4-x}{5}$
(3) $\frac{x-5}{4}$ (4) इनमें से कोई नहीं

3. For any operation*, be defined on θ as a*b = $\frac{a+b}{3}$,then 1 * 2 =

(1) 3 (2) 1
(4) 0 (4) none of these

कोई संक्रिया *, θ पर a*b = $\frac{a+b}{3}$ से परिभाषित है तो 1 * 2 =

(1) 3 (2) 1
(4) 0 (4) इनमें से कोई नहीं

4. **Principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is**

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ का मुख्य मान है

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{4}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

5. **$\cos-1\left(\frac{1-x^2}{1+x^2}\right) =$**

(1) $2\cos^{-1}x$ (2) $2\sin^{-1}x$

(3) $2\tan^{-1}x$ (4) $\cos^{-1}(2x)$

6. **If $A=\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$, then $3A =$**

(1) $\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$ (2) $\begin{bmatrix}1 & 6\\ 9 & 12\end{bmatrix}$

(3) $\begin{bmatrix}3 & 6\\ 9 & 18\end{bmatrix}$ (4) none of these

यदि $A=\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$, तो $3A =$

(1) $\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$ (2) $\begin{bmatrix}1 & 6\\ 9 & 12\end{bmatrix}$

(3) $\begin{bmatrix}3 & 6\\ 9 & 18\end{bmatrix}$ (4) इनमें से कोई नहीं

7. **If A and B are two square matrices, then $(A+B)^T =$**

(1) $A^T - B^T$ (2) $A^T + B^T$

(3) $2A^T$ (4) $A^T - 4B^T$

यदि A और B दो वर्ग आव्यूह हैं, तो $(A+B)^T=$

(1) $A^T - B^T$ (2) $A^T + B^T$

(3) $2A^T$ (4) $A^T - 4B^T$

8. **If $\begin{bmatrix}x+3 & 2x\\ 6 & y\end{bmatrix}=\begin{bmatrix}7 & 8\\ 6 & 3\end{bmatrix}$, then values of x and y are**

(1) $x = 4, y = 3$ (2) $x = 3, y = 4$

(3) $x = 3, y = 3$ (4) none of these

यदि $\begin{bmatrix}x+3 & 2x\\ 6 & y\end{bmatrix}=\begin{bmatrix}7 & 8\\ 6 & 3\end{bmatrix}$, तो x तथा y के मान हैं

(1) $x = 4, y = 3$ (2) $x = 3, y = 4$

(3) $x = 3, y = 3$ (4) इनमें से कोई नहीं

9. $\begin{vmatrix}1 & 1 & 1\\ x+y & y+z & z+x\\ z & x & y\end{vmatrix} =$

(1) 0 (2) 1

(3) −1 (4) $x+y+z$

10. $\begin{vmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{vmatrix} =$

(1) $\cos\theta$ (2) $\sin\theta$

(3) 1 (4) 0

11. **$\frac{d}{dx}(\tan x^2) =$**

(1) $\sec^2(x^2)$ (2) $2x\sec^2(x^2)$

(3) $2x$ (4) none of these

$\frac{d}{dx}(\tan x^2) =$

(1) $\sec^2(x^2)$ (2) $2x\sec^2(x^2)$

(3) $2x$ (4) इनमें से कोई नहीं

12. **$\frac{d}{dx}(\sin^{-1}x) =$**

(1) $\frac{1}{\sqrt{1-x^2}}$ (2) $\frac{-1}{\sqrt{1-x^2}}$

(3) $\frac{1}{1+x^2}$ (4) none of these

$\frac{d}{dx}(\sin^{-1}x) =$

(1) $\frac{1}{\sqrt{1-x^2}}$ (2) $\frac{-1}{\sqrt{1-x^2}}$

(3) $\frac{1}{1+x^2}$ (4) इनमें से कोई नहीं

13. **$\frac{d}{dx}(\log \sec x)=$**

(1) $\sec x$ (2) $\tan x$

(3) $\cot x$ (4) none of these

$\frac{d}{dx}(\log \sec x)=$

(1) $\sec x$ (2) $\tan x$

(3) $\cot x$ (4) इनमें से कोई नहीं

14. **if $x = \alpha\cos\theta$, $y = \alpha\sin\theta$, then $\frac{dy}{dx} =$**

(1) $\tan\theta$ (2) $\cot\theta$

(3) $-\tan\theta$ (4) none of these

यदि $x = \alpha\cos\theta$, $y = \alpha\sin\theta$, तो $\frac{dy}{dx} =$

(1) $\tan\theta$ (2) $\cot\theta$

(3) $-\tan\theta$ (4) इनमें से कोई नहीं

15. **If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx} =$**

(1) $\frac{y}{x}$ (2) $\frac{x}{y}$

(3) $-\frac{x}{y}$ (4) none of these

यदि $x^m y^n = (x+y)^{m+n}$, तो $\frac{dy}{dx} =$

(1) $\frac{y}{x}$ (2) $\frac{x}{y}$

(3) $-\frac{x}{y}$ (4) इनमें से कोई नहीं

16. **The slope of the curve $y^2 = x$ at the point (1, 1) is**

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$

(3) 1 (4) None of these

वक्र $y^2 = x$ की ढ़ाल (1, 1) बिन्दु पर है

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$

(3) 1 (4) इनमें से कोई नहीं

17. **The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is**

(1) 3 (2) $-\frac{1}{3}$

(3) −3 (4) none of these

वक्र $y = 2x^2 + 3\sin x$ के $x = 0$ पर अभिलम्ब पर ढ़ाल है

(1) 3 (2) $-\frac{1}{3}$

(3) −3 (4) इनमें से कोई नहीं

18. $\int \sec^2 x . dx =$

(1) $\tan x + c$ (2) $\tan 2x + c$

(3) $\cot x$ (4) none of these

$\int \sec^2 x . dx =$

(1) $\tan x + c$ (2) $\tan 2x + c$

(3) $\cot x$ (4) इनमें से कोई नहीं

19. $\int \frac{\text{sex}^2 x}{\text{cosec}^2 x} dx =$

(1) $x - \tan x + c$ (2) $\tan x - x + c$

(3) $-\tan x - x + c$ (4) none of these

$\int \frac{\text{sex}^2 x}{\text{cosec}^2 x} dx =$

(1) $x - \tan x + c$ (2) $\tan x - x + c$

(3) $-\tan x - x + c$ (4) इनमें से कोई नहीं

20. $\int \frac{dx}{9 + x^2} =$

(1) $\tan -1\left(\frac{x}{3}\right) + c$ (2) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$

(3) $3\tan^{-1}\left(\frac{x}{3}\right) + c$ (4) none of these

$\int \frac{dx}{9 + x^2} =$

(1) $\tan -1\left(\frac{x}{3}\right) + c$ (2) $\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$

(3) $3\tan^{-1}\left(\frac{x}{3}\right) + c$ (4) इनमें से कोई नहीं

21. $\int \frac{dx}{\sqrt{16 - x^2}} =$

(1) $\sin^{-1}\left(\frac{x}{4}\right) + c$ (2) $\log|x + \sqrt{16 - x^2}|$

(3) $\log|x + \sqrt{x^2 + 16}|$ (4) none of these

$\int \frac{dx}{\sqrt{16 - x^2}} =$

(1) $\sin^{-1}\left(\frac{x}{4}\right) + c$ (2) $\log|x + \sqrt{16 - x^2}|$

(3) $\log|x + \sqrt{x^2 + 16}|$ (4) इनमें से कोई नहीं

22. **the radius of a circle is increasing at the rate of 0.3 cm/sec. The rate of increase of its perimeter is**

(1) 0.4π cm/sec (2) 0.6π cm/sec

(3) 0.8π cm/sec (4) none of these

किसी वृत्त की त्रिज्या की वृद्धि दर 0.3cm/sec हो तो इसकी परिधि की वृद्धि दर होगी

(1) 0.4π cm/sec (2) 0.6π cm/sec

(3) 0.8π cm/sec (4) इनमें से कोई नहीं

23. $\int_a^b x^2 . dx =$

(1) $\frac{b^3 - a^3}{3}$ (2) $\frac{a^3 - b^3}{3}$

(3) $\frac{a^4 - b^4}{4}$ (4) none of these

$\int_a^b x^2 . dx =$

(1) $\frac{b^3 - a^3}{3}$ (2) $\frac{a^3 - b^3}{3}$

(3) $\frac{a^4 - b^4}{4}$ (4) इनमें से कोई नहीं

24. $\int_1^{\sqrt{3}} \frac{dx}{1 + x^2} =$

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$

(3) $\frac{\pi}{12}$ (4) none of these

$\int_1^{\sqrt{3}} \frac{dx}{1 + x^2} =$

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$

(3) $\frac{\pi}{12}$ (4) इनमें से कोई नहीं

25. $\int_{-\pi/2}^{\pi/2} \sin^9 x dx =$

(1) -1 (2) 1

(3) 0 (4) none of these

$\int_{-\pi/2}^{\pi/2} \sin^9 x dx =$

(1) -1 (2) 1

(3) 0 (4) इनमें से कोई नहीं

26. **The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 2\left(\frac{dy}{dx}\right)^3 + 9y = 0$ is**

(1) 2 (2) 4

(3) 1 (4) none of these

अवकल समीकरण $\left(\frac{d^2y}{dx^2}\right)^2 + 2\left(\frac{dy}{dx}\right)^3 + 9y = 0$ की कोटी है

(1) 2 (2) 4

(3) 1 (4) इनमें से कोई नहीं

27. **The integrating factor of the differential equation $\frac{dy}{dx} + y\sec^2 x = \tan x . \sec^2 x$ is**

(1) $\tan x$ (2) $e^{\tan x}$

(3) $\log \tan x$ (4) none of these

अवकल समीकरण $\frac{dy}{dx} + y\sec^2 x = \tan x . \sec^2 x$ का समाकलन गुणांक है

(1) $\tan x$ (2) $e^{\tan x}$

(3) $\log \tan x$ (4) इनमें से कोई नहीं

28. **If $x\hat{i} + 2\hat{j} = 3\hat{i} - y\hat{j}$, then find (x, y).**

(1) 3, −2 (2) 2, 3

(3) −3, 2 (4) none of these

यदि $x\hat{i} + 2\hat{j} = 3\hat{i} - y\hat{j}$, तो (x, y) ज्ञात कीजिए।

(1) 3, −2 (2) 2, 3

(3) −3, 2 (4) इनमें से कोई नहीं

29. **The position vector of the point (1, 0, 2) is**

(1) $\hat{i} + 2\hat{k}$ (2) $\hat{i} + 2\hat{j}$

(3) $\hat{i} + 3\hat{j} + 2\hat{k}$ (4) $\hat{i} + \hat{j} + 2\hat{k}$

बिन्दु (1, 0, 2) का स्थिति सदिश है

(1) $\hat{i} + 2\hat{k}$ (2) $\hat{i} + 2\hat{j}$

(3) $\hat{i} + 3\hat{j} + 2\hat{k}$ (4) $\hat{i} + \hat{j} + 2\hat{k}$

30. **The scalar dot product of vector $5\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} - 4\hat{j} + 7\hat{k}$ is**

(1) 10 (2) −10

(3) 15 (4) none of these

$5\hat{i} + \hat{j} - 3\hat{k}$ और $3\hat{i} - 4\hat{j} + 7\hat{k}$ का अदिश गुणनफल है।

(1) 10 (2) −10

(3) 15 (4) इनमें से कोई नहीं

31. $\hat{i} \times \hat{j} =$

(1) 0 (2) $\hat{k}$

(3) $-\hat{k}$ (4) none of these

$\hat{i} \times \hat{j} =$

(1) 0 (2) $\hat{k}$

(3) $-\hat{k}$ (4) इनमें से कोई नहीं

32. $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) =$

(1) $2(\vec{a} \times \vec{b})$ (2) $\vec{a} \times \vec{b}$

(3) $|\vec{a}|^2 - |\vec{b}|^2$ (4) none of these

$(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) =$

(1) $2(\vec{a} \times \vec{b})$ (2) $\vec{a} \times \vec{b}$

(3) $|\vec{a}|^2 - |\vec{b}|^2$ (4) इनमें से कोई नहीं

33. $\vec{a}.\vec{a} =$

(1) 0 (2) 1

(3) $|\vec{a}|^2$ (4) none of these

$\vec{a}.\vec{a} =$

(1) 0 (2) 1

(3) $|\vec{a}|^2$ (4) इनमें से कोई नहीं

34. **The direction cosines of z-axis are**

(1) (0, 0, 0) (2) (1, 0, 0)

(3) (0, 1, 0) (4) (0, 0, 1)

z-अक्ष दिक् कोज्याएँ हैं

(1) (0, 0, 0) (2) (1, 0, 0)

(3) (0, 1, 0) (4) (0, 0, 1)

35. **Find the equation of the line joining (−2, 4, 2) and (7, −2, 5).**

(1) $\frac{x}{-2} = \frac{y}{4} = \frac{z}{2}$

(2) $\frac{x}{7} = \frac{y}{-2} = \frac{z}{5}$

(3) $\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z-2}{1}$

(4) None of these

बिन्दु (−2, 4, 2) और (7, −2, 5) को मिलाने वाली रेखा समीकरण होगा

(1) $\frac{x}{-2} = \frac{y}{4} = \frac{z}{2}$

(2) $\frac{x}{7} = \frac{y}{-2} = \frac{z}{5}$

(3) $\frac{x+2}{3} = \frac{y-4}{-2} = \frac{z-2}{1}$

(4) इनमें से कोई नहीं

36. Line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ passes through the point.

(1) (5, −4, 6) (2) (5, 4, −6)
(3) (4, 5, 6) (4) none of these

रेखा $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$ बिन्दु .... से गुजरती है।

(1) (5, −4, 6) (2) (5, 4, −6)
(3) (4, 5, 6) (4) इनमें से कोई नहीं

37. The direction cosines of the normal to the plane 2x − 3y − 6z − 3 = 0

(1) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (2) $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$
(3) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (4) none of these

समतल 2x − 3y − 6z − 3 = 0 के अभिलम्ब की दिक् कोज्याएँ हैं

(1) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (2) $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$
(3) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (4) इनमें से कोई नहीं

38. Find the perpendicular distance of the plane $2x + y - 2z + 1 = 0$ from the point (0, −1, 3).

(1) $2\sqrt{3}$ (2) $\frac{2}{3}$
(3) 2 (4) None of these

बिन्दु (0, −1, 3) से तल $2x + y - 2z + 1 = 0$ की लम्बवत दूरी ज्ञात कीजिए।

(1) $2\sqrt{3}$ (2) $\frac{2}{3}$
(3) 2 (4) इनमें से कोई नहीं

39. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{4}$, then $P\left(\frac{A}{B}\right) =$

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$
(3) $\frac{2}{3}$ (4) none of these

यदि $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ तथा $P(A \cap B) = \frac{1}{4}$, तो $P\left(\frac{A}{B}\right) =$

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$
(3) $\frac{2}{3}$ (4) इनमें से कोई नहीं

40. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$, then $P(A \cup B) =$

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$
(3) $\frac{1}{2}$ (4) none of these

यदि $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{3}$ तथा $P(A \cap B) = \frac{1}{4}$, तो $P(A \cup B) =$

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$
(3) $\frac{1}{2}$ (4) इनमें से कोई नहीं

Ans. (1)- 2, (2)- 1, (3)- 2, (4)- 3, (5)- 2, (6)- 3, (7)- 2, (8)- 1, (9)- 1, (10)- 3, (11)- 2, (12)- 1, (13)- 2, (14)- 3, (15)- 1, (16)- 1, (17)- 2, (18)- 1, (19)- 2, (20)- 2, (21)- 1, (22)- 2, (23)- 1, (24)- 3, (25)- 3, (26)- 1, (27)- 2, (28)- 1, (29)- 1, (30)- 2, (31)- 2, (32)- 1, (33)- 3, (34)- 4, (35)- 3, (36)- 1, (37)- 1, 3, (38)- 3, (39)- 2, (40)- 4

झारखंड अधिविद्य परिषद्
ANNUAL INTERMEDIATE EXAMINATION 2023

गणित

हल प्रश्न-पत्र / SOLVED PAPER

विषयनिष्ठ प्रश्नोत्तर

INSTRUCTIONS / निर्देश :

1. Examinees are required to answer in their own words as far as practicable.

   परीक्षार्थी यथासंभव अपने शब्दों में ही उत्तर दें।

2. This question paper has three sections : **A, B** and **C**. Total number of questions is **19**.

   इस प्रश्नपत्र में तीन खण्ड – **A, B** एवं **C** हैं। कुल प्रश्नों की संख्या **19** है।

3. **Section-A** - Question Nos. **1 - 7** are very short answer type. Answer any five of these questions. Each question carries 2 marks.

   खण्ड–A में प्रश्न संख्या **1 - 7 अति लघु उत्तरीय प्रकार** के हैं। इनमें से किन्हीं **पाँच** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर दीजिए। प्रत्येक प्रश्न की अधिमानता 2 अंक निर्धारित है।

4. **Section-B** - Question Nos. **8 - 14** are Short answer type. Answer any five of these questions. Each question carries **3** marks.

   खण्ड–B – प्रश्न संख्या **8 - 14** लघु उत्तरीय हैं। इनमें से किन्हीं **पाँच** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर दीजिए। प्रत्येक प्रश्न की अधिमानता **3** अंक निर्धारित है।

5. **Section-C** - Question Nos. **15 - 19** are Long answer type. Answer any three of these questions. Each question carries **5** marks. **खण्ड–C** – प्रश्न संख्या **15 - 19** दीर्घ उत्तरीय हैं। इनमें से किन्हीं **तीन** प्रश्नों के उत्तर दीजिए। प्रत्येक प्रश्न का उत्तर दीजिए। प्रत्येक प्रश्न की अधिमानता **5** अंक निर्धारित है।

Section - A

खण्ड–A

(Very short answer type questions)

(अति लघु उत्तरीय प्रश्न)

Answer any five questions.

किन्हीं पाँच प्रश्नों के उत्तर दें।

1. If $f : R \rightarrow R$ and $g : R \rightarrow R$ are defind by $f(x) = \sqrt{x}$ *and* $g(x) = x^2$, **then find f o g(x).**

   यदि $f : R \rightarrow R$ तथा $g : R \rightarrow R$, $f(x) = \sqrt{x}$ तथा $g(x) = x^2$, से परिभाषित हैं, तो f o g(x) ज्ञात कीजिए।

2. Prove that $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$.

   सिद्ध करें कि $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$.

3. If $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -4 \\ 1 & -3 \\ 4 & 0 \end{bmatrix}$, then find AB.

   यदि $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 2 \end{bmatrix}$ तथा $B = \begin{bmatrix} 2 & -4 \\ 1 & -3 \\ 4 & 0 \end{bmatrix}$, तो AB ज्ञात कीजिए।

4. If $y = x^{\sin x}$, find $\frac{dy}{dx}$.

   यदि $y = x^{\sin x}$, $\frac{dy}{dx}$ निकालें।

5. Find $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$.

   ज्ञात करें $\int \frac{\cos x}{\sqrt{1+\sin x}} dx$.

6. Solve the following differential equation.

   $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

   निम्नलिखित अवकल समीकरण को हल करें:

   $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

7. Compute $P\left(\frac{A}{B}\right)$ if P(B) = 0.5 and P(A∩B) = 0.32.

   $P\left(\frac{A}{B}\right)$ ज्ञात कीजिए यदि P(B) = 0.5 तथा P(A∩B) = 0.32.

Section - B
खण्ड – B
(Short answer type questions)
(लघु उत्तरीय प्रश्न)

Answer any five questions

किन्हीं **पाँच** प्रश्नों का उत्तर दें।

8. Prove that

$$\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix} = (3x+\lambda).\lambda^2.$$

सिद्ध कीजिए कि

$$\begin{vmatrix} x+\lambda & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda \end{vmatrix} = (3x+\lambda).\lambda^2.$$

9. If $f(x) = 2x + 5$, when $x \le 2$
$= 2x - 5$ When $x > 2$
then test the continuity of $f(x)$ at $x = 2$.

यदि $f(x) = 2x + 5$, जब $x \le 2$
$= 2x - 5$, जब $x > 2$
तो $x = 2$ पर $f(x)$ की संतता की जाँच करें।

10. Find the interval in which the function $f(x) = x^2 + 2x - 5$ is strictly increasing or strictly decreasing.

अंतराल ज्ञात कीजिए जिसमें फलन $f(x) = x^2 + 2x - 5$ निरंतर वर्धमान या निरंतर ह्रासमान है।

11. Evaluate $\int \frac{2x+1}{x^2+4x-3}dx$.

ज्ञात कीजिए $\int \frac{2x+1}{x^2+4x-3}dx$.

12. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$.

ज्ञात कीजिए $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$.

13. Find the area of a region bounded by the curve $y^2 = 4x$ and the straight line $x = 3$.

वक्र $y^2 = 4x$ तथा सरल रेखा $x = 3$ से घिरे क्षेत्र का क्षेत्रफल ज्ञात कीजिए।

14. Find the angle between the following pair of lines:
$\vec{r} = (3\hat{i}+2\hat{j}-4\hat{k})+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ and
$\vec{r} = (5\hat{i}-2\hat{j}+\hat{k})+t(3\hat{i}+2\hat{j}+6\hat{k})$.

निम्नलिखित रेखायुग्मों के बीच का कोण ज्ञात कीजिए:
$\vec{r} = (3\hat{i}+2\hat{j}-4\hat{k})+\lambda(\hat{i}+2\hat{j}+2\hat{k})$ तथा
$\vec{r} = (5\hat{i}-2\hat{j}+\hat{k})+t(3\hat{i}+2\hat{j}+6\hat{k})$.

Section - C
खण्ड – C
(Long answer type questions)
(दीर्घ उत्तरीय प्रश्न)

Answer any three questions.

किन्हीं तीन प्रश्नों के उत्तर दें।

15. Solve the system of the linear equations, using matrix method:
$3x - y + z = 5, 2x - 2y + 3z = 7, x + y - z = -1.$

रैखिक समीकरण निकाय को आव्यूह विधि से हल करें:

$3x - y + z = 5, 2x - 2y + 3z = 7, x + y - z = -1.$

16. Find the maximum and minimum values of the given function:
$f(x) = x^3 - 6x^2 + 9x + 5.$

दिए गए फलन का महत्तम एवं न्यूनतम मान ज्ञात कीजिए:

$f(x) = x^3 - 6x^2 + 9x + 5.$

17. Find the shortest distance between the lines
$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+4\hat{k})$ and
$\vec{r} = (2\hat{i}+4\hat{j}+5\hat{k})+\lambda(3\hat{i}+4\hat{j}+5\hat{k})$

रेखाओं के बीच न्यूनतम दूरी ज्ञात कीजिए
$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+4\hat{k})$ तथा
$\vec{r} = (2\hat{i}+4\hat{j}+5\hat{k})+\lambda(3\hat{i}+4\hat{j}+5\hat{k})$

18. Solve the LPP graphically :
Maximize $Z = 4x + y$
Subject to constraints.
$x + y \le 5, 3x + y \le 9, x \ge 0, y \ge 0.$

रैखिक प्रोग्रामन समस्या को आलेखीय विधि से हल करें :
अधिकतमीकरण कीजिए $Z = 4x + y$
जबकि व्यवरोध
$x + y \le 5, 3x + y \le 9, x \ge 0, y \ge 0.$

19. A man is known to speak truth 3 times out of 5 times. He throws a die and reports that it is one (1) on the die. Find the the probability that it is actually one (1) on the die.

एक व्यक्ति के बारे में यह ज्ञात है कि वह 5 बार में 3 बार सत्य बोलता है। वह एक पासे को उछालता है और बताता है कि पासे पर आनेवाली संख्या एक (1) है। इसकी प्रायिकता ज्ञात कीजिए कि पासे पर आनेवाली संख्या वास्तव में एक (1) है।

Solutions (Subjective)

Section - A

खण्ड – A

1. $fog(x) = f[g(x)] = f(x^2) = \sqrt{x^2} = x$.

2. $L.H.S = \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$$= \tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}} = \tan^{-1}\frac{\frac{48+77}{264}}{\frac{264-14}{264}}$$

$$= \tan^{-1}\frac{125}{250} = \tan^{-1}\frac{1}{2} = R.H.S.$$

3. $$AB = \begin{bmatrix}2 & 3 & 5\\4 & 6 & 2\end{bmatrix}\begin{bmatrix}2 & -4\\1 & -3\\4 & 0\end{bmatrix}$$

$$= \begin{bmatrix}4+3+20 & -8-9+0\\8+6+8 & -16-18+0\end{bmatrix}$$

$$= \begin{bmatrix}27 & -17\\22 & -34\end{bmatrix}.$$

4. $\because y = x^{\sin x}$

taking log on both sides, we get

$\log y = \log x^{\sin x} = \sin x . \log x$,

differentiating both sides w.r.to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}.\sin x + \cos x.\log x$$

$$\therefore \frac{dy}{dx} = y\left[\frac{\sin x}{x} + \cos x.\log x\right]$$

$$= x^{\sin x}\left[\frac{\sin x}{x} + \cos x.\log x\right].$$

5. Put $1 + \sin x = t$

$\therefore \cos x dx = dt$

$$\therefore I = \int\frac{dt}{\sqrt{t}} = \int t^{-1/2}dt$$

$$= \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c.$$

$$= 2\sqrt{t} + c = 2\sqrt{1+\sin x} + c.$$

6. $\because \frac{dy}{dx} = \frac{1+x^2}{1+y^2}$

$\therefore (1 + y^2)dy = (1 + x^2)dx$

integrating both sides, we get

$$\int(1+y^2)dy = \int(1+x^2)dx$$

$$\Rightarrow y + \frac{y^3}{3} = x + \frac{x^3}{3} + c$$

Which is the required general solution.

7. $\therefore P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{32}{50} = \frac{16}{25}$.

Section - B

8. $$\Delta = \begin{vmatrix}x+\lambda & x & x\\x & x+\lambda+ & x\\x & x & x+\lambda\end{vmatrix}$$

By $c_1 \to c_1 + c_2 + c_3$, we have

$$\Delta = \begin{vmatrix}3x+\lambda & x & x\\3x+\lambda & x+\lambda & x\\3x+\lambda & x & x+\lambda\end{vmatrix}$$

$$= (3x+\lambda)\begin{vmatrix}1 & x & x\\1 & x+\lambda & x\\1 & x & x+\lambda\end{vmatrix}$$

By $R_1 \to R_1 - R_2$ and $R_3 \to R_3 - R_2$, We get

$$\Delta = (3x+\lambda)\begin{vmatrix}0 & -\lambda & 0\\1 & x+\lambda & x\\0 & -\lambda & \lambda\end{vmatrix}$$

Resolving along R_1, We get

$$\Delta = (3x+\lambda)\lambda\begin{vmatrix}1 & x\\0 & \lambda\end{vmatrix}$$

$$= (3x + \lambda)\,\lambda\,(\lambda - 0) = (3x + \lambda)\,.\,\lambda^2$$

$\therefore \Delta = (3x + \lambda) . \lambda^2$. Proved.

9. We have,

$$L.H.limit = \lim_{h\to 0} f(2-h)$$

$$= \lim_{h\to 0}\{2(2-h)+5\} = 9.$$

$$R.H. limit = \lim_{h\to 0} f(2+h)$$

$$= \lim_{h\to 0}\{2(2+h)-5\} = -1.$$

since L.H. limit $\neq$ R.H. limit

$\therefore f(x)$ is not continuous at $x = 2$.

10. We have $f(x) = x^2 + 2x - 5$

$\therefore f'(x) = 2x + 2$

$f(x)$ is strictly increasing if $f'(x) > 0$

$\Rightarrow 2x + 2 > 0 \Rightarrow x > -1$

i.e $x \in (-1, \infty)$

f(x) is strictly decreasing if $f'(x) < 0$

$\Rightarrow 2x + 2 < 0 \Rightarrow x < -1$

i.e $x \in (-\infty, -1)$

hence f(x) is strictly increasing on $(-1, \infty)$ and strictly decreasing on $(-\infty, -1)$.

11. $I = \int \frac{2x+1}{x^2+4x-3}dx.$

$= \int \frac{2x+4-3}{x^2+4x-3}dx$

$= \int \frac{2x+4}{x^2+4x-3}dx - \int \frac{3}{x^2+4x-3}dx$

$= I_1 - I_2$

Where $I_1 = \int \frac{2x+4}{x^2+4x-3}dx$

and $I_2 = \int \frac{3}{x^2+4x-3}dx$

Now, $I_1 = \int \frac{2x+4}{x^2+4x-3}dx$

Put $x^2+4x-3 = t$

$\therefore (2x+4)dx = dt$

$\therefore I_1 = \int \frac{dt}{t} = \log|t| + c_1$

$= \log|x^2+4x-3| + c_1$

and $I_2 = \int \frac{3}{x^2+4x-3}dx = \int \frac{3}{x^2+4x+4-7}dx$

$= \int \frac{3}{(x+2)^2-(\sqrt{7})^2}dx = \frac{3}{2\sqrt{7}}\log\left|\frac{x+2-\sqrt{7}}{x+2+\sqrt{7}}\right| + c_2$

$\therefore I = I_1 - I_2$

$= \log|x^2+4x-3| + c_1 - \frac{3}{2\sqrt{7}}\log\left|\frac{x+2-\sqrt{7}}{x+2+\sqrt{7}}\right| - c_2$

$= \log|x^2+4x-3| - \frac{3}{2\sqrt{7}}\log\left|\frac{x+2-\sqrt{7}}{x+2+\sqrt{7}}\right| + c$

Where $c_1 - c_2 = c$.

12. $I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$.........(i)

$= \int_0^{\pi/2} \frac{\sqrt{\tan(\frac{\pi}{2}-x)}}{\sqrt{\tan(\frac{\pi}{2}-x)}+\sqrt{\cot(\frac{\pi}{2}-x)}}dx$

$I = \int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx$.........(ii)

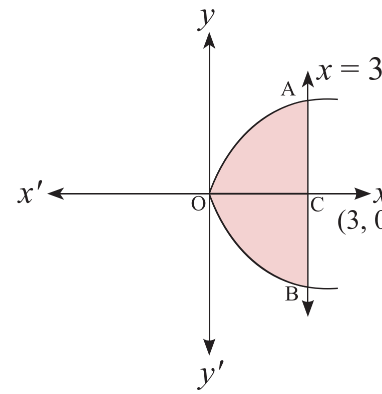
Adding (i) and (ii), we get

$2I = \int_0^{\pi/2} \frac{\sqrt{\tan x}+\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_0^{\pi/2} 1dx.$

$= [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$.

13.



Given curve is $y^2 = 4x$ ....(i)

and line is $x = 3$............(ii)

Curve (i) is a parabola with vertex (0, 0), axis x-axis.

$\therefore$ Required area = 2(area of region OACO)

$= 2\int_0^3 y dx = 2\int_0^3 \sqrt{4x}\,dx$

$= 4\int_0^3 \sqrt{x}\,dx = 4 \times \frac{2}{3}[x^{3/2}]_0^3$

$= \frac{8}{3} \times (3)^{3/2} = 8 \times 3^{3/2-1} = 8 \times 3^{1/2}$

$= 8\sqrt{3}$ sq.unit

14. We have $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$

and $\vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

Let θ be the angle between them.

$\therefore \cos\theta = \left|\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}\right|$

$= \left|\frac{3+4+12}{\sqrt{1^2+2^2+2^2}+\sqrt{3^2+2^2+6^2}}\right|$

$= \frac{19}{3 \times 7} = \frac{19}{21}$

$\theta = \cos^{-1}\frac{19}{21}$.

Section - C

15. Given equations are

$3x - y + z = 5$ ....(i)

$2x - 2y + 3z = 7$ ....(ii)

and $x + y - z = -1$ ....(iii)

Matrix form of the system of equation is Ax = B , Where

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 2 & -2 & 3 \\ 1 & 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$$

$$\text{Now } |A| = \begin{vmatrix} 3 & -1 & 1 \\ 2 & -2 & 3 \\ 1 & 1 & -1 \end{vmatrix}$$

$=3(2-3)+1(-2-3)+1(2+2)$

$= 3(-1) + 1(-5) + 1(4) = -3 -5 + 4 = -4 \neq 0$

$\therefore$ matrix A is non-singular. Here A^{-1} exists.

Now, Cofactors of elements of |A| are,

$$A_{11} = \begin{vmatrix} -2 & 3 \\ 1 & -1 \end{vmatrix} = -1, A_{12} = -\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = 5, A_{13} = \begin{vmatrix} 2 & -2 \\ 1 & 1 \end{vmatrix} = 4$$

$$A_{21} = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = 0, A_{22} = \begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = -4, A_{23} = \begin{vmatrix} 3 & -1 \\ 1 & 1 \end{vmatrix} = -4$$

$$A_{31} = \begin{vmatrix} -1 & 1 \\ -2 & 3 \end{vmatrix} = -1, A_{32} = -\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = -7, A_{33} = \begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix} = -4$$

$$\therefore \text{adj} = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -1 & 0 & -1 \\ 5 & -4 & -7 \\ 4 & -4 & -4 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj}A = \frac{-1}{4}\begin{bmatrix} -1 & 0 & -1 \\ 5 & -4 & -7 \\ 4 & -4 & -4 \end{bmatrix}$$

$\therefore x = A^{-1}B.$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -1 & 0 & -1 \\ 5 & -4 & -7 \\ 4 & -4 & -4 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$$

$$= -\frac{1}{4}\begin{bmatrix} -5+0+1 \\ 25-28+7 \\ 20-28+4 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} -4 \\ 4 \\ -4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \Rightarrow x = 1, y = -1, z = 1.$$

16. Given function is $f(x) = x^3 - 6x^2 + 9x + 5$

$\therefore f'(x) = 3x^2 - 12x + 9$..............(i)

$\therefore$ For maximum or minimum value of $f(x)$, $f'(x) = 0$

$\Rightarrow 3x^2 - 12x + 9 = 0$

or, $x^2 - 4x + 3 = 0$

or, $(x-1)(x-3) = 0 \Rightarrow x = 1$ or 3

$\therefore$ from (i), $f''(x) = 6x - 12$

$\therefore f''(1) = 6 \times 1 - 12 = -6 < 0$

$\therefore f(x)$ is maximum at $x = 1$

$\therefore$ Maximum value $= (1)^3 - 6(1)^2 + 9.1 + 5$

$=1 - 6 + 9 + 5 = 9$

and $f''(3) = 6 \times 3 - 12 = 6 > 0$

$\therefore f(x)$ is minimum at $x = 3$

$\therefore$ minimum value $= (3)^3 - 6(3)^2 + 9 \times 3 + 5$

$= 27 - 54 + 27 + 5 = 5.$

17. Given lines are

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} + 4\hat{k})$...............(i)

and $\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 5\hat{k})$................(ii)

Here $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Now, $\vec{a_2} - \vec{a_1} = (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \hat{i} + 2\hat{j} + 2\hat{k}$

$$\text{and } \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$= \hat{i}(15-16) - \hat{j}(10-12) + \hat{k}(8-9)$

$= -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore$ Shortest distance between (i) and (ii) is

$$d = \left| \frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

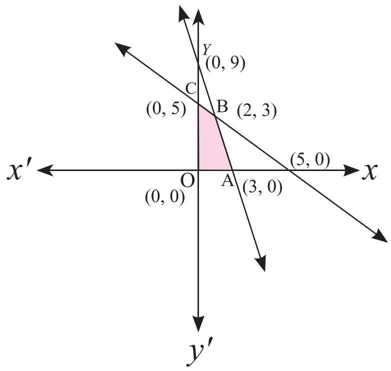
$$= \left| \frac{(\hat{i} + 2\hat{j} + 2\hat{k}).(-\hat{i} + 2\hat{j} - \hat{k})}{\sqrt{1+4+1}} \right|$$

$$= \left| \frac{-1+4-2}{\sqrt{6}} \right| = \frac{1}{\sqrt{6}} \text{ units.}$$

18. First of all we draw the graph of lines.

$x + y = 5$...............(i)

$3x + y = 9$ ....(ii)



Shaded region OABCD is the feasible region, Which is bounded. Its vertices are O(0, 0), A(3, 0), B(2, 3) and C(0, 5).

Now, $z = 4x + y$.

$\therefore$ At 0(0, 0), $z = 4 \times 0 + 0 = 0$

At A(3, 0), $z = 4 \times 3 + 0 = 12$

At B(2, 3), $z = 4 \times 2 + 3 = 11$

At C(0, 5), $z = 4 \times 0 + 5 = 5$

$\therefore$ Clearly, z is maximum at A(3, 0) and maximum value is 12.

19. Let A = Man reports that 1 comes on die.

Let A_1 = Event of getting 1 on the die.

and A_2 = Event of not getting 1 on the die.

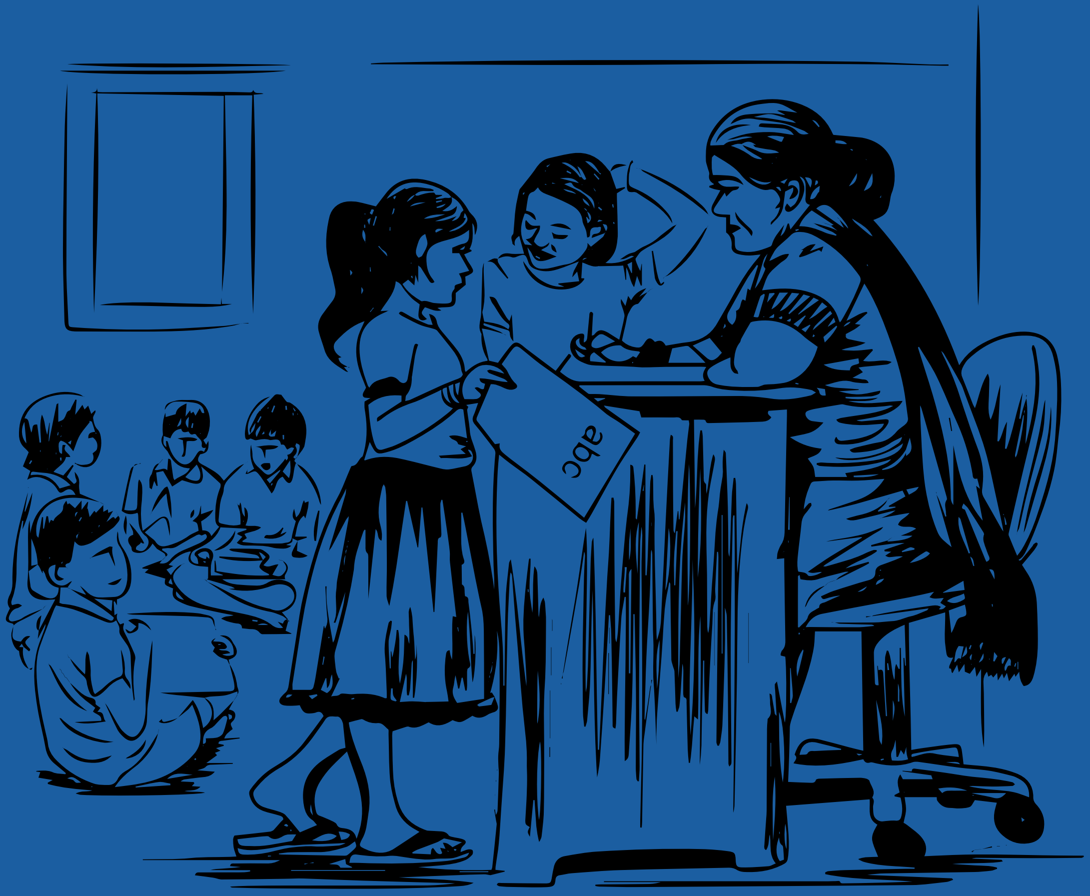
$\therefore P(A_1) = \frac{1}{6}$ and $P(A_2) = \frac{5}{6}$

Now, $P(A/A_1) = \frac{3}{5}$ and $P(A/A_2) = \frac{2}{5}$

$\therefore$ By Baye's theorem,

$$P(A_1/A) = \frac{P(A_1).P(A/A_1)}{P(A_1).P(A/A_1) + P(A_2).P(A/A_2)}$$

$$\frac{\frac{1}{6} \times \frac{3}{5}}{\frac{1}{6} \times \frac{3}{5} + \frac{5}{6} \times \frac{2}{5}} = \frac{\frac{3}{30}}{\frac{13}{30}} = \frac{3}{13}.$$



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